

# THE DEFINABILITY OF ALMOST DISJOINT FAMILIES AND LONG WELL-ORDERS AT HIGHER CARDINALS

PHILIPP LÜCKE

ABSTRACT. Given a cardinal  $\kappa$  that is a limit of measurable cardinals, we study collections of subsets of  $\kappa$  that are definable by  $\Sigma_1$ -formulas that only use the cardinal  $\kappa$  and bounded subsets of  $\kappa$  as parameters. Starting from a *perfect subset theorem* for these sets, we generalize two classical non-definability results to higher cardinals. First, we show that a classical result of Mathias on the complexity of *maximal almost disjoint families* of sets of natural numbers can be generalized to Ramsey limits of measurables. Second, we prove that for a limit of countably many measurable cardinals, the existence of a simply definable well-ordering of subsets of  $\kappa$  of length at least  $\kappa^+$  implies the existence of a projective well-ordering of the reals.

This is joint work in progress with Sandra Müller (Vienna).