PARTITION PROPERTIES AND DEFINABILITY

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Abstract. A Jónsson algebra is an algebra with no proper subalgebras of the same cardinality, i.e., a set A equipped with countably many functions from finite products of A to A that has the property that no proper subset of the same cardinality is closed under all these functions. It is easy to see that there is a countably infinite Jónsson algebra. Moreover, if there exists a Jónsson algebra of some infinite cardinality κ, then there exists such an algebra of the next higher cardinality κ^+. In contrast, the question whether there is a Jónsson algebra whose cardinality is equal to the first limit cardinal $\aleph_\omega$ is a long-standing open problem in set theory that motivated central developments in this field. In my talk, I want to present a new approach to restrict the class of models of set theory in which no Jónsson algebras of size $\aleph_\omega$ exist that is based on an analysis of set-theoretic definability at this cardinal. This is joint work in progress with Omer Ben-Neria (Jerusalem).