The Apery algorithm for a plane singularity

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This is a joint work with V. Barucci and R. Froberg. Let k be an algebraically closed field of characteristic zero and let $R = k[[X_1, \ldots, X_n]]/I$ be a one dimensional reduced ring. Since the integral closure $\overline{R}$ of R in its total ring of fractions is of the form $\overline{R} = k[[t_1]] \times \cdots \times k[[t_d]]$ (where d is the number of minimal primes of R), it is possible to associate to R a value semigroup $v(R) \subseteq \mathbf{N}^d$.

The study of this kind of rings is connected with the study of curve singularities and the use of the value semigroup is a classical method to get information on the ring.

In case of plane singularities (i.e. $R = k[[X, Y]]/(f)$) it is possible to find a strict connection between the value semigroup and the multiplicity tree of the ring. In particular, when R and R' are both local, it is possible to determine the value semigroup of the blowing up R' of R by the value semigroup of R and, viceversa, to determine $v(R)$ by $v(R')$ and by the multiplicity of R (this fact in the one branch case ($d = 1$) is a classical result of Apery). Using this result, for $d = 1, 2$, we can give an algorithmic criterion to check if a subsemigroup of $\mathbf{N}^d$ is the value semigroup of a plane singularity with d branches and, conversely, we can construct $v(R)$ by the multiplicity tree of the singularity. The study of the case $d \geq 3$ is in progress.