Singularity exchange at infinity

M. Tibar

How does the topology change in a family of complex polynomials $f_s : \mathbf{C}^n \to \mathbf{C}$? One has at hand the highly developed local theory of deformations of singularities, but how to use such methods and what else is needed in order to treat our more global problem? Our study focuses on the new phenomena which occur in such a context. One of the novelties turns out to be the singularity exchange at infinity, which can be described as follows: as s tends to say 0, some isolated singularities of f_s tend to infinity and disappear from $\mathbf{C}^n$, producing in the same time "virtual" singularities of f_0. Even if not situated in $\mathbf{C}^n$, those virtual singularities manifest in the topology of f_0. The total quantity of singularity involved in this phenomenon may not be conserved. Inspite of the fact that some of the ingredients do not behave well in deformations, we show how to prove semi-continuity results which enable one to find rules of the exchange phenomenon.