Comparison theorems in logarithmic cohomology

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Given a variety defined over the complex numbers (even singular), the problem of defining a suitable De Rham cohomology and its comparison with other cohomologies was posed several years ago by many authors (see for example Grothendieck, Hartshorne, Deligne...). This has led (among other reasons) to the definition of a infinitesimal sites and finally to the comparison between the Algebraic De Rham Cohomology of a singular scheme, its Infinitesimal (or Crystalline) Cohomology, and the Betti Cohomology of its associated analytic space (see works of Grothendieck, Hartshorne, Deligne, Herrera-Lieberman, Illusie, Berthelot, Ogus, and others).

In this talk, we propose a generalization of these results and definitions to the logarithmic setting (characteristic zero, over the complex numbers), inspired by works of K. Kato, C. Nakayama, L. Illusie, A. Ogus, A. Shiho. Namely, given $S = \operatorname{Spec} \mathbb{C}$ (endowed with the trivial log structure) and an fs not necessary (ideally) log smooth log scheme Y over S, we analyze the connection between the Log De Rham Cohomology of Y, its Log Infinitesimal Cohomology $H^{\bullet}(Y_{inf}^{log}, \mathcal{O}_{Y_{inf}^{log}})$, and its Log Betti Cohomology, which is defined as the Cohomology of its associated Kato-Nakayama topological space Y_{log}^{an} . We first show that they are isomorphic under the hypothesis that there exists an exact closed immersion of Y into a log smooth log scheme X over S, then we extend these comparison theorems to any log scheme over S, by working with good embedding systems, log formal tubes and by using descent properties. These results generalize the (ideally) log smooth log case developed by Kato-Nakayama.

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