Topological Machine Learning Seminar

Computational topology inside the REXASIPRO project

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The REXASIPRO project

The REXASIPRO project

REXASI-PRO | Reliable & eXplAinable Swarm Intelligence for People with Reduced mObility – CL4-Human-01-01 HE Project Proposal

HORIZON EUROPE | CALL HORIZON-CL4-2021-HUMAN-01-01

Verifiable robustness, energy efficiency and transparency for Trustworthy AI: Scientific excellence boosting industrial competitiveness

Type of action: RIA – Innovation Action

Proposal Budget € million: 4 (100% for all)

European Commission

The REXASIPRO project

REXASI-PRO| CAN WE TRUST IN AI?

Artificial Intelligent (AI) become omnipresent in our society: autonomous cars, flying taxi, robots, medicine discovery, etc.... **However, society is hesitant about it** (known as blade runner/terminator effect)

AI Perceived as a Black Box

Future is to develop public trust into AI solution: Transparent, Safe, Secure, Reliable & Green AI

Forbes

In AI (Can) We Trust?

The REXASIPRO project

REXASIPRO | Demonstration

In **REXASIPRO** we **will develop** a new **framework** in which **safety**, **security** and **explainability** are entangled for the development of a **Trustworthy Artificial Swarm Intelligence** solution. The framework will **enable** the **collaboration** among a swarm composed by **autonomous wheelchairs and flying-robots to enable a seamless door-to-door experience for people with reduced mobility**.

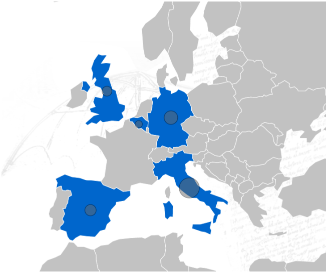
Deutsches Forschungszentrum für Künstliche Intelligenz GmbH

HOVERING solutions

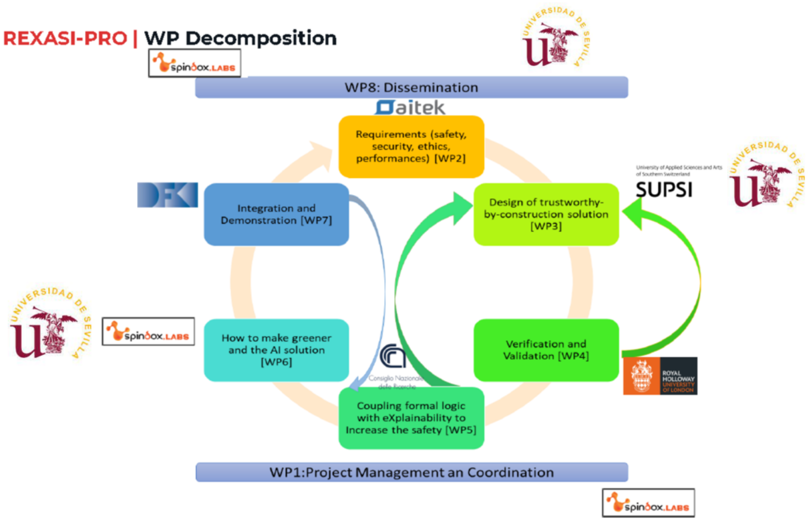
The REXASIPRO project

REXASI-PRO | Partners

Participant No. *	Participant organisation name
1 (Coordinator)	Spindox Labs
2	Italian National Council of Research
3	Deutsches Forschungszentrum für Künstliche Intelligenz
4	Dalle Molle Institute for Artificial Intelligence
5	ROYAL HOLLOWAY AND BEDFORD NEW COLLEGE
6	V-Research
7	AITEK
8	UNIVERSIDAD DE SEVILLA
9	Hovering Solution
10	EURONET
11(Subcontracting)	Scuola di Robotica (Ethics)



The REXASIPRO project



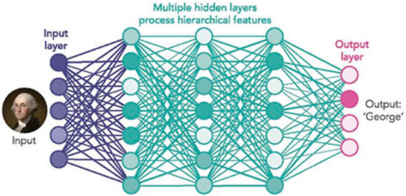
Artificial intelligence and climate change

Natural language processing (NLP)

$\simeq$

5 cars' CO2 emissions throughout their useful life

Deep neural networks (DNN)



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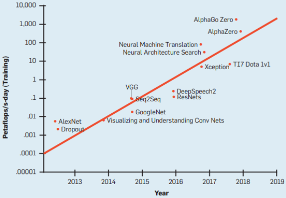

Computational topology inside REXASIPRO

Goals

Goals

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Main goal

Use computational topology to design new methods to achieve **green artificial intelligence models.**

Example of AI model: neural networks

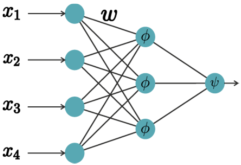
Given $d, k > 0$, a multi-layer feed-forward neural network defined between spaces $X \subseteq \mathbb{R}^d$ and $Y \subseteq \mathbb{R}^k$ is a function $\boldsymbol{F}: X \to Y$ composed of $m+1$ functions:

$$\boldsymbol{F} = f_{m+1} \circ f_m \circ \cdots \circ f_1$$

where the integer $m > 0$ is the number of hidden layers and, for $i \in \{1, \ldots, m+1\}$, the function $f_i: X_{i-1} \to X_i$ is defined as

$$f_i(y) := \phi_i(\omega^{(i)}; y; b_i)$$

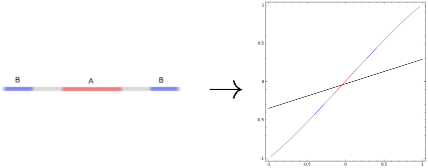
where ϕ_i is the activation function, b_i the vector of the bias term, and $\omega^{(i)}$ the matrix of weights.



Goals

Why topology?

- Topology studies properties of spaces that are preserved against **continuous deformations.**
- Neural networks are compositions of **continuous functions.**
- The training process consists of **continuous deformations.** The network *deforms* space so that data of different classes are separable by a hyperplane.



- Real-world high-dimensional data sets actually lie in low-dimensional **manifolds** (manifold hypothesis).

Goals

Main goal

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Specific Goals

O.1. Reduce the input dataset.

O.2. Create synthetic samples that can quickly train a model.

O.3. Build optimized models based on topology.

O.4. Simplify the model preserving its learning capacity.

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O.1. Reducing the input dataset

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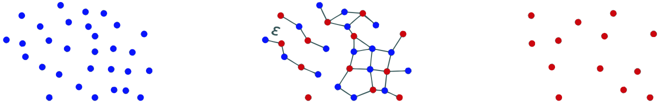
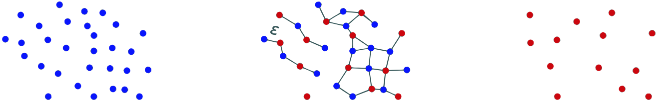
Representative datasets



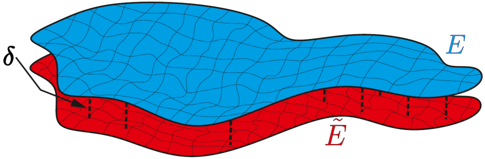
O.1. Reducing the input dataset

Representative datasets



Theorem (with E Paluzo-Hidalgo, MA Gutiérrez-Naranjo. 2022)

Let $\tilde{\mathscr{D}} = \{(x, c_x) : x \in X \subset \mathbb{R}^n\}$ be a λ-balanced ε-representative dataset of the binary dataset $\mathscr{D}$. Let $\mathscr{N}_\omega$ be a perceptron with weights $\omega \in \mathbb{R}^{n+1}$. Then, $|\mathbb{E}(\omega, \mathscr{D}) - \mathbb{E}(\omega, \tilde{\mathscr{D}})| \leq \frac{43||\omega||_*}{54}\varepsilon = \delta$, where $\mathbb{E}(\omega, \mathscr{D}) = \frac{1}{|X|}\sum(c_x - \mathscr{N}_w(x))$.



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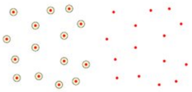
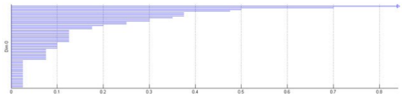
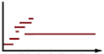
Theorem (with E Paluzo-Hidalgo, MA Gutiérrez-Naranjo. 2022)

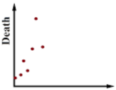
Let $\tilde{\mathscr{D}} \subset \mathbb{R}^n$ be a λ-balanced ε-representative dataset of the binary dataset $\mathscr{D}$. Then, $\frac{1}{2} d_B\big(B(\mathscr{D}), B(\tilde{\mathscr{D}})\big) \leq \varepsilon$.

Persistent homology

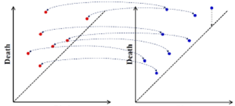








$d_B\big(B(\mathscr{D}), B(\tilde{\mathscr{D}})\big)$



O.1. Reducing the input dataset

Persistent homology $\Rightarrow$persistent entropy

Definition (with H Chintakunta, MJ Jimenez, H Krim. 2015)

$H(B) = \sum \frac{\ell_i}{L} \cdot \log\left(\frac{\ell_i}{L}\right)$ where L is the sum of the lengths ℓ_i of the bars.

* $0 < H(B) \leq log(n)$ being n the amount of bars.
* $H(B) = log(n)$ when all bars have the same length.

Theorem (with N Atienza, M Soriano-Trigueros. 2020)

Under mild assumptions, $|H(B(\mathscr{D})) - H(B(\tilde{\mathscr{D}}))| \leq k(n, L) \cdot \varepsilon$.

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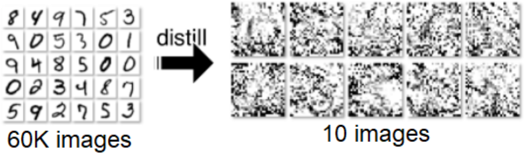

Dataset Distillation

Tongzhou Wang[1,2] Jun-Yan Zhu[2]
Antonio Torralba[2] Alexei A. Efros[3]

[1]Facebook AI Research [2]MIT CSAIL [3]UC Berkeley

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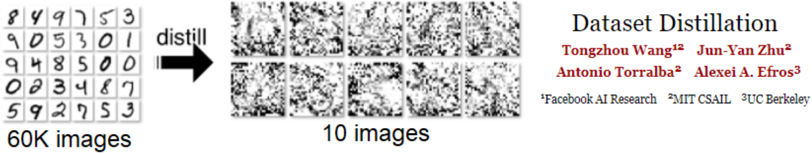
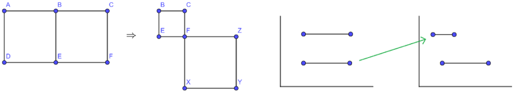


Partial Matchings



$f : U \Rightarrow V$ $\qquad$ $\mathscr{M}_f : B(U) \to B(V)$

O.2. Creating synthetic samples

Partial matchings

$\mathscr{M}_f : B(U) \times B(V) \to \mathbb{Z}$ is defined by

$$\mathscr{M}_f((a,b),(c,d)) = \dim \left(\varinjlim_{t \in (a,b) \cap (c,d)} X_{(a,b)(c,d)t} \right)$$

Theorem (with M Soriano-Trigueros, A. Torras. Submitted)

$\mathscr{M}_f$ is well-defined, linear and can be computed using matrix row reductions.

$$M^f_{(a,b)(c,d)t} := \left(\begin{array}{c|cc} & A^-_{(a,b)t} & A^+_{(a,b)t} \setminus A^-_{(a,b)t} \\ \hline B^-_{(c,d)t} & \multicolumn{2}{c}{\text{Ignored rows}} \\ B^+_{(c,d)t} \setminus B^-_{(c,d)t} & \text{Block 1} & \text{Block 2} \\ B_t \setminus B^+_{(c,d)} & * & * \end{array} \right)$$

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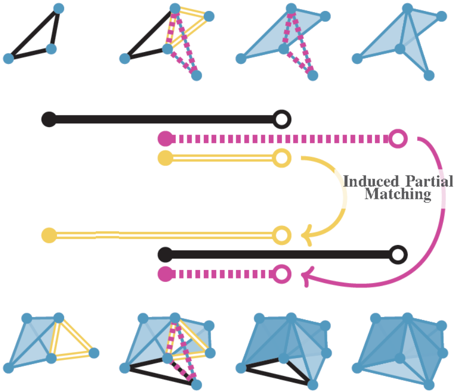

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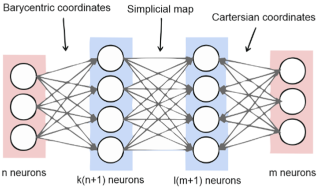
Simplicial map neural network (SMNN).



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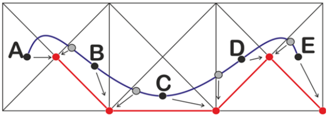

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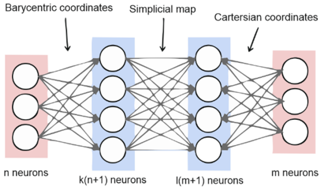

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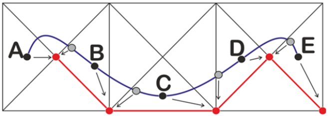

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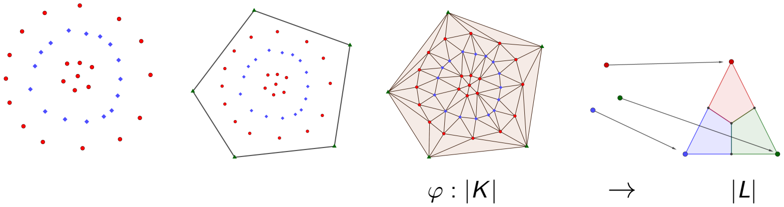

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$\mathcal{N}_\varphi$ correctly classifies $\mathcal{D}$.

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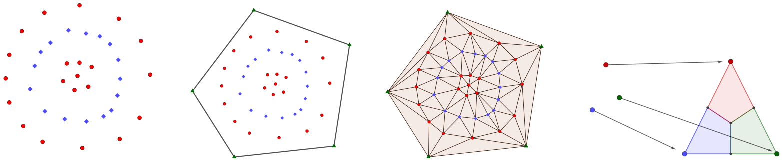

$$\varphi : |K| \rightarrow |L|$$

Theorem (with E Paluzo-Hidalgo, MA Gutiérrez-Naranjo, J Heras. 2021)

$\mathscr{N}_\varphi$ *correctly classifies* $\mathscr{D}$.

O.4. Simplifying the model

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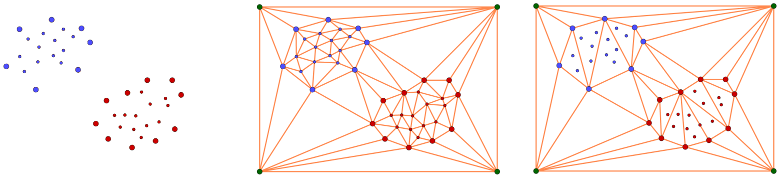
Optimized SMNN



$\mathscr{D}$ $\mathscr{N}_{\varphi}$ $\mathscr{N}_{\tilde{\varphi}}$

Theorem (with E Paluzo-Hidalgo, MA Gutiérrez-Naranjo, J Heras. 2022)

Both $\mathscr{N}_{\varphi}$ and $\mathscr{N}_{\tilde{\varphi}}$ correctly classifies $\mathscr{D}$.

O.4. Simplifying the model

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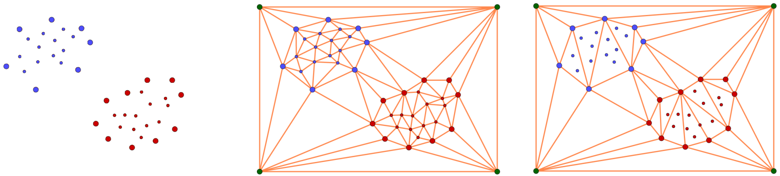

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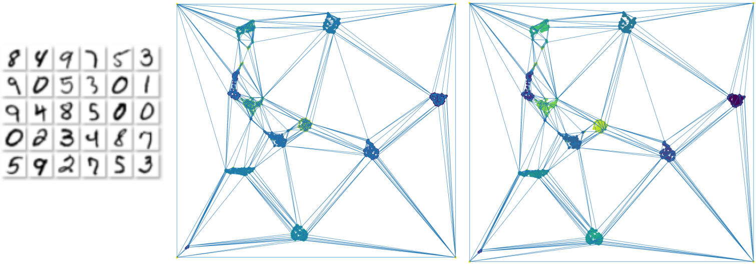

	Dataset Size	2-Simplices
Original	1801	3596
Reduced	604	305

Future research lines

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- Formally prove that representative sets maintain accuracy in deep neural networks and more general AI models.
- Construct morphism-induced partial matches between more general persistence modules.
- Use the SMNNs as a tool for the explainability, reliability and transparency of an artificial intelligence model (trustworthy AI).
- Create more efficient and robust variants of SMNNs.

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Bibliography

Bibliography

Bibliography

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Moltes gràcies per la vostra atenció !