Graph-Based Segmentation Methods.

From a Classical Approach to Graph Convolutional Networks

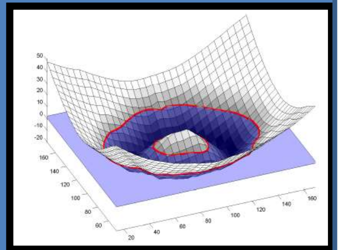
Dr Esmeralda Ruiz Pujadas; 18th October 2022

euCanSHare T5.5

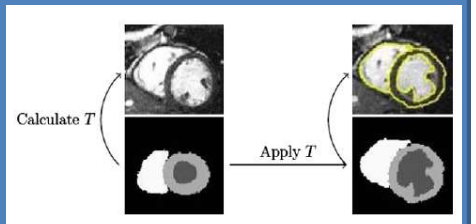
1 – Review of Traditional Segmentation Approaches

Review of Classical Segmentation Methods

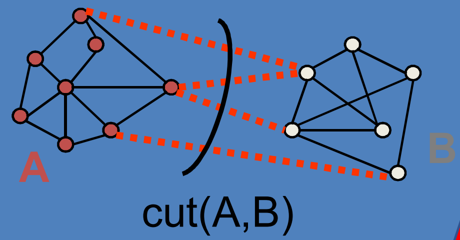
Snake: Deformation of curves according to the minimization of an energy functional



Level Sets: The deforming contour is considered as a zero level of a higher dimensional function



Atlas: Register the labeled atlas onto the image and apply the obtained transformation to the atlas



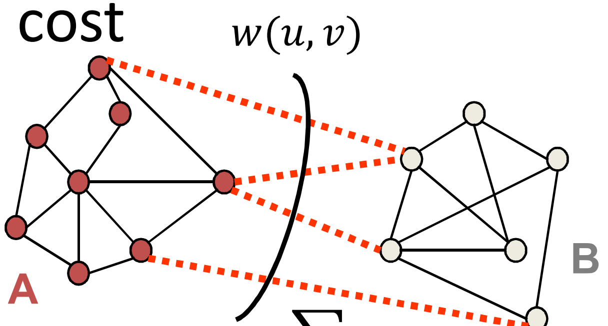
Graph Cuts: An image represented as a graph can be partitioned in 2 disjoint sets removing edges connecting the two parts

Active Shape Models: Model describes objects as a linear combination of eigenvectors around the mean shape. The model parameters which best fit the target image are estimated giving the final segmentation

2 – Introduction to Graph-based Segmentation Methods

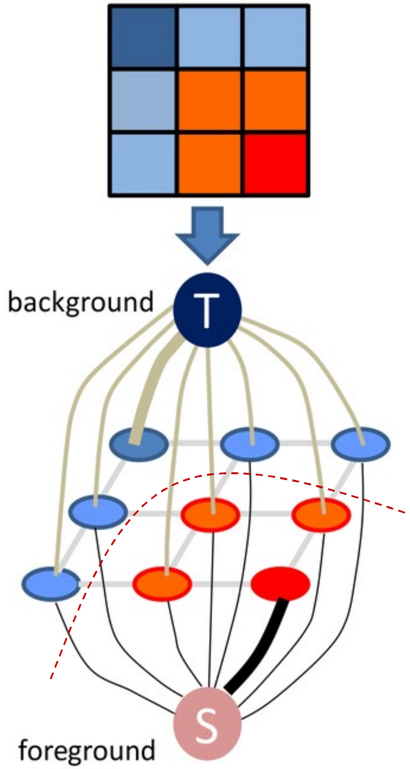
Graph Cuts

- Image is represented as a graph:
  - Pixels: nodes
  - Edges: weights indicating similarity
- A graph can be partitioned in 2 disjoint sets removing edges connecting the two parts (red dotted line)
- Optimal bi-partitioning is the one that minimizes the cut cost



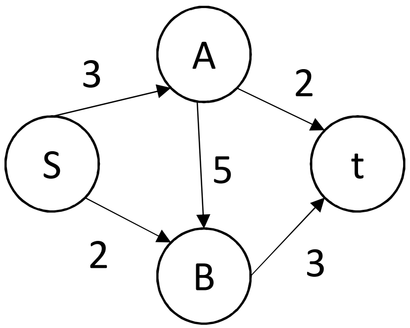
$$cut(A,B) = \sum_{u \in A, v \in B} w(u,v)$$

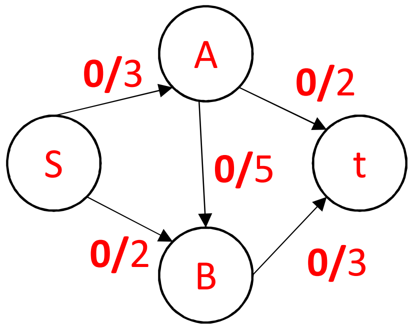
Ford-Fulkerson Method



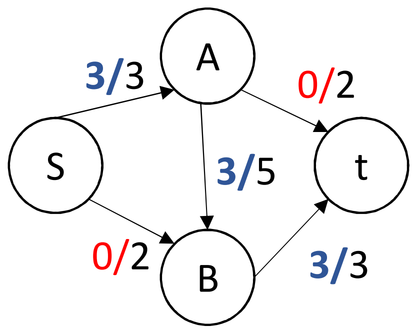
[Shiyan Pang et al., Sensors, 2018]

Ford-Fulkerson Method

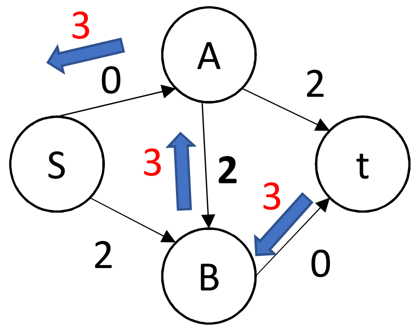




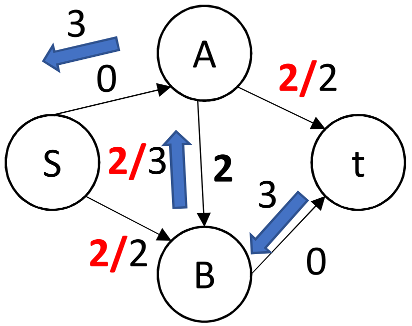
Initialization



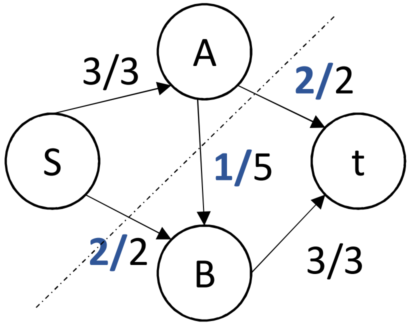
Flow



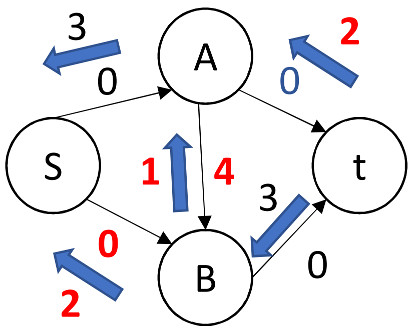
Residual Graph



Find the path in the Residual Graph



Update the flow



Update the Residual Graph

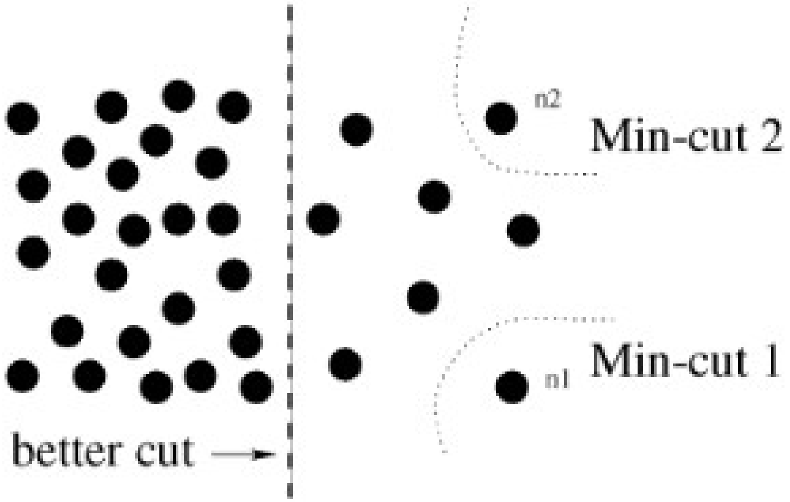
The maximum outgoing flow from the node t is 5 !

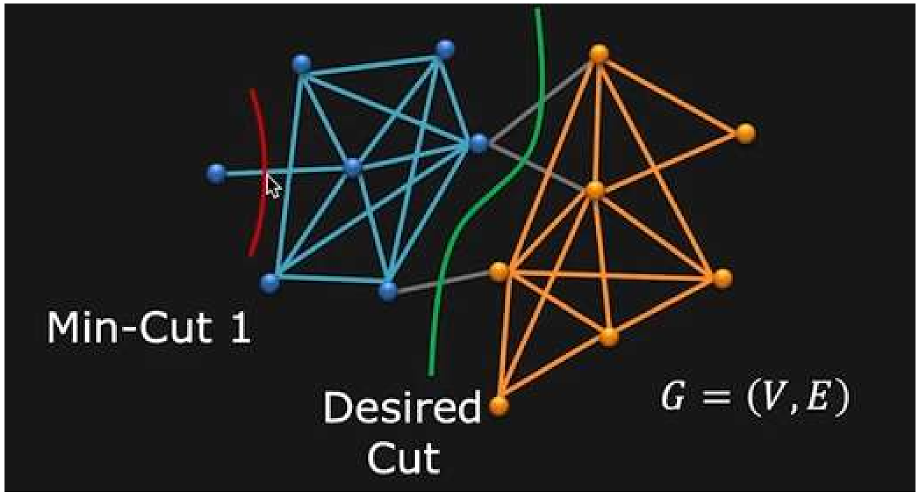
Minimum cut of this graph = 5

[T.Donwey, Donwey webpage, 2019]

Drawbacks of Graph Cuts

- It can be biased toward producing a small contour.

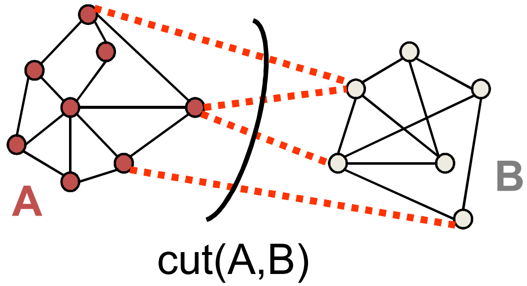




[Shi and Malik, PAMI, 2000 ;Shree Nayar, First Principles of CV, 2021]

Normalized cuts

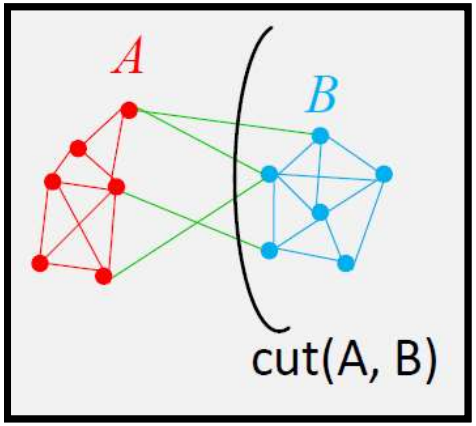
- Normalized Cuts is a graph-based image segmentation method
  - Systematic global energy minimization
  - Robust to initialization
  - No seeds required
  - Limited number of parameters



- Limitation: Incorporation of shape prior is challenging
  - The formulation must be changed

Normalized Cuts Partition

- An image transformed into a graph:
  - Nodes: Voxels of the image
  - Edges: Weights (W) between voxels.
- Balance between:
  - Cost of the boundary
  - Size of regions
- Optimal bi-partitioning is the one that minimizes the cut cost [Shi and Malik, PAMI, 2000]

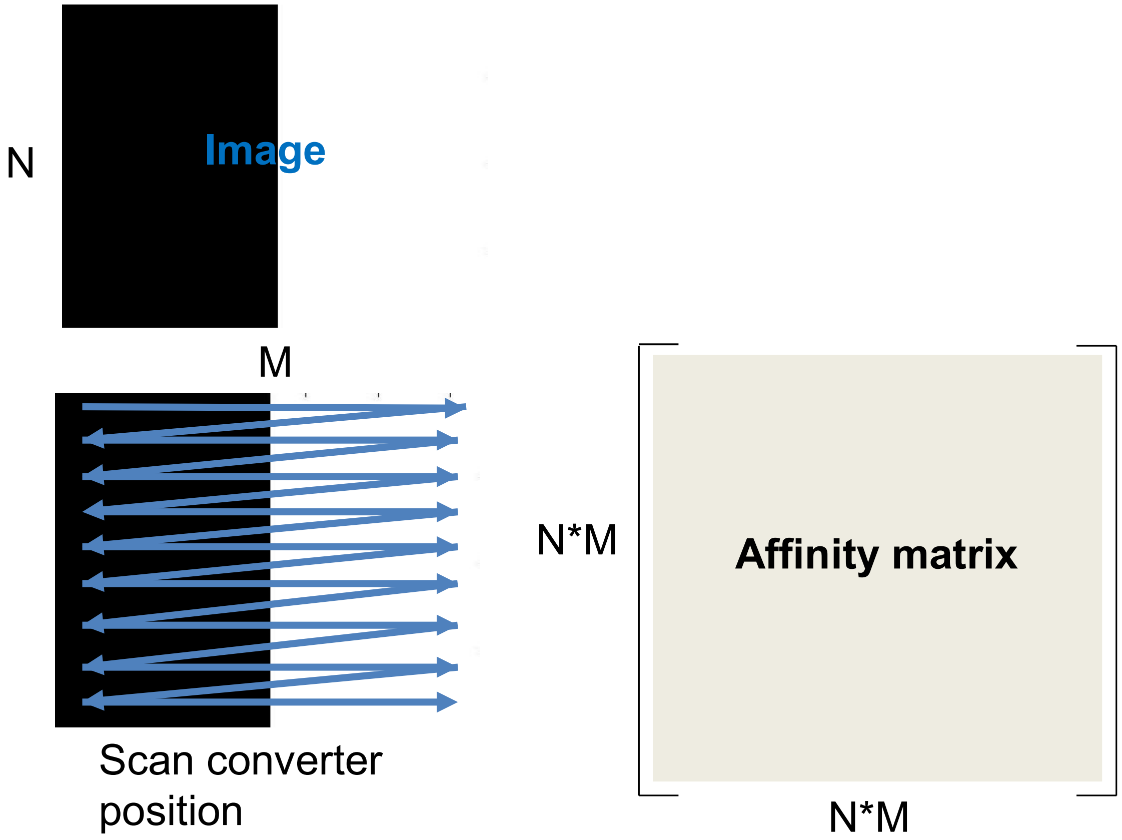


$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(B,A)}{assoc(B,V)} \Rightarrow min_{x} Ncut(\boldsymbol{x}) = min_{\boldsymbol{y}} \frac{\boldsymbol{y}^T(\mathbf{D}-\mathbf{W})\boldsymbol{y}}{\boldsymbol{y}^T\mathbf{D}\boldsymbol{y}} \Rightarrow (\mathbf{D}-\mathbf{W})\boldsymbol{y} = \lambda \mathbf{D}\boldsymbol{y}.$$

assoc(A,V) are the total weights of edges connecting nodes within A. (Equiv. to B)

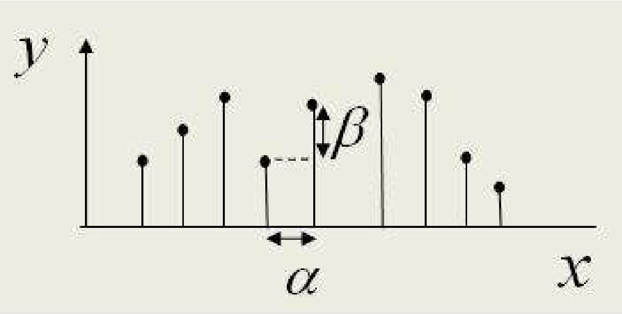
Normalized Cuts Algorithm: Affinity Matrix

- Construct the Affinity matrix (W) as follows:



Weights of Affinity Matrix

- Calculate the weights as follows:

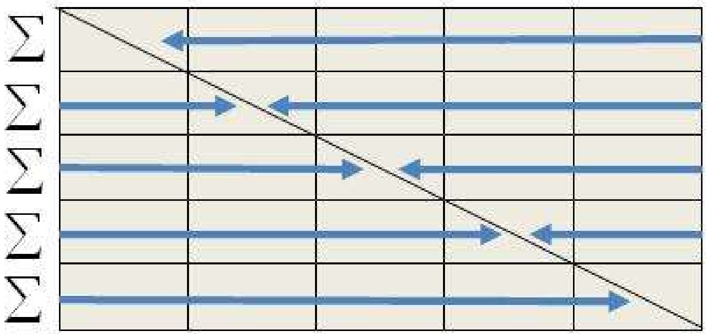


  - Product of distances of spatial and intensity Gaussians

$$W_{ij} = G_{\sigma_x}(\alpha) * G_{\sigma_y}(\beta)$$

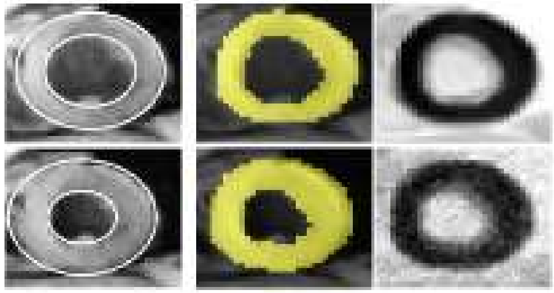
Eigenvector Decomposition

- Diagonal matrix (D):



Weight matrix

- Eigenvector decomposition:
  - Laplacian Matrix = (D-W)
- Choose the eigenvector with the second smallest eigenvalue
- Back-project to the image
- Thresholding



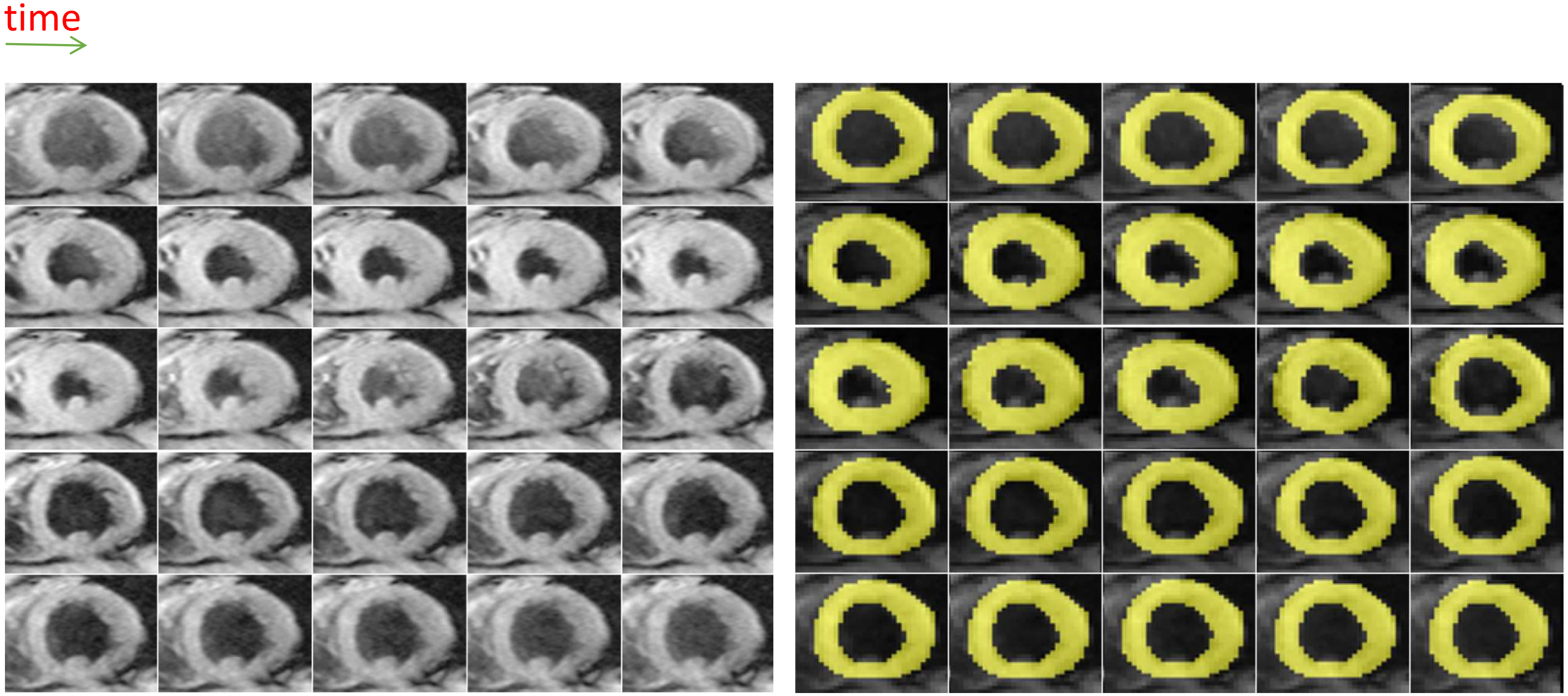
(a) (b)

(a) Original image with single prior. (b) The proposed method and its normalized cut.

[E.Ruiz et.al, TIP, 2013]

Proposed Method: Example

- 25 cardiac MRI images of one volunteer over 1 ECG cycle



[E.Ruiz et.al, TIP, 2013]

Proposed Method: Example

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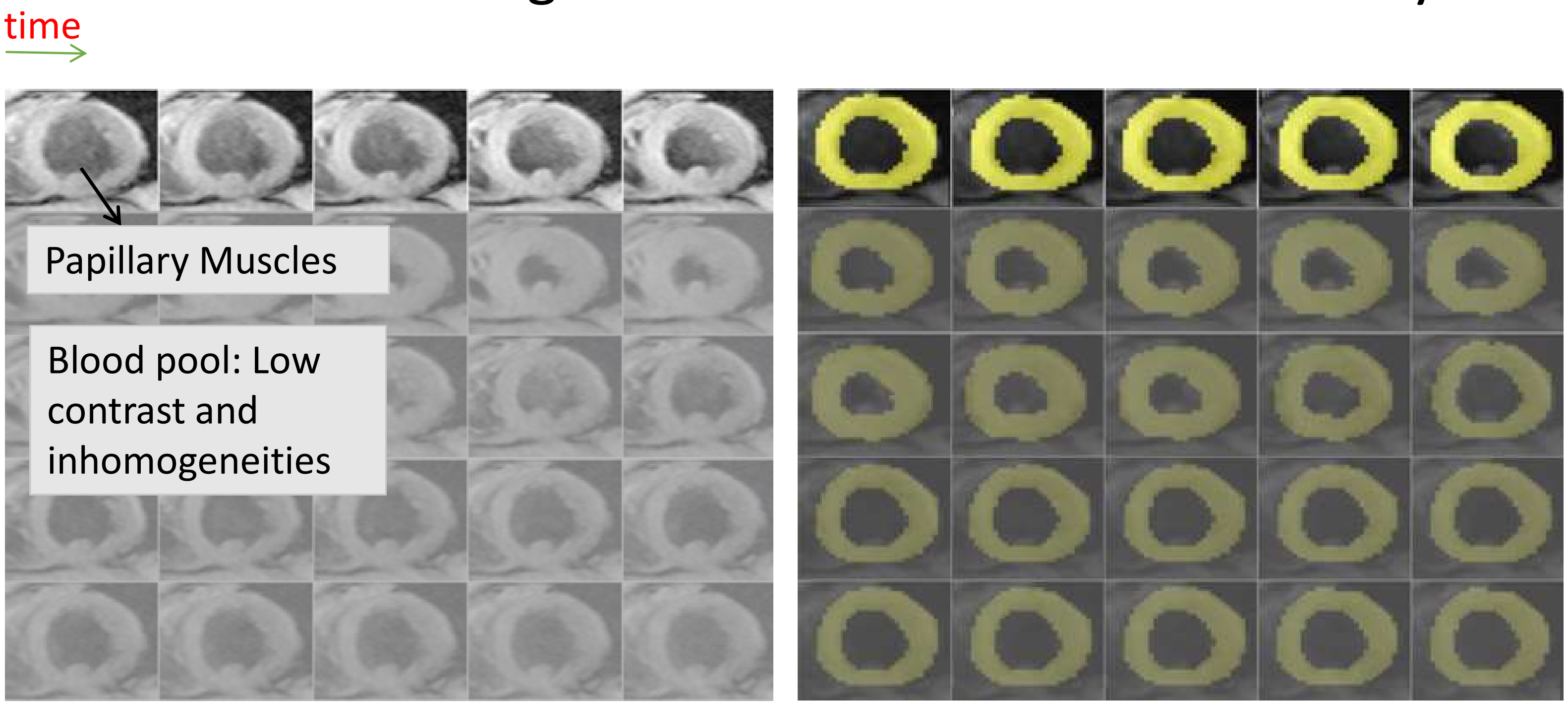

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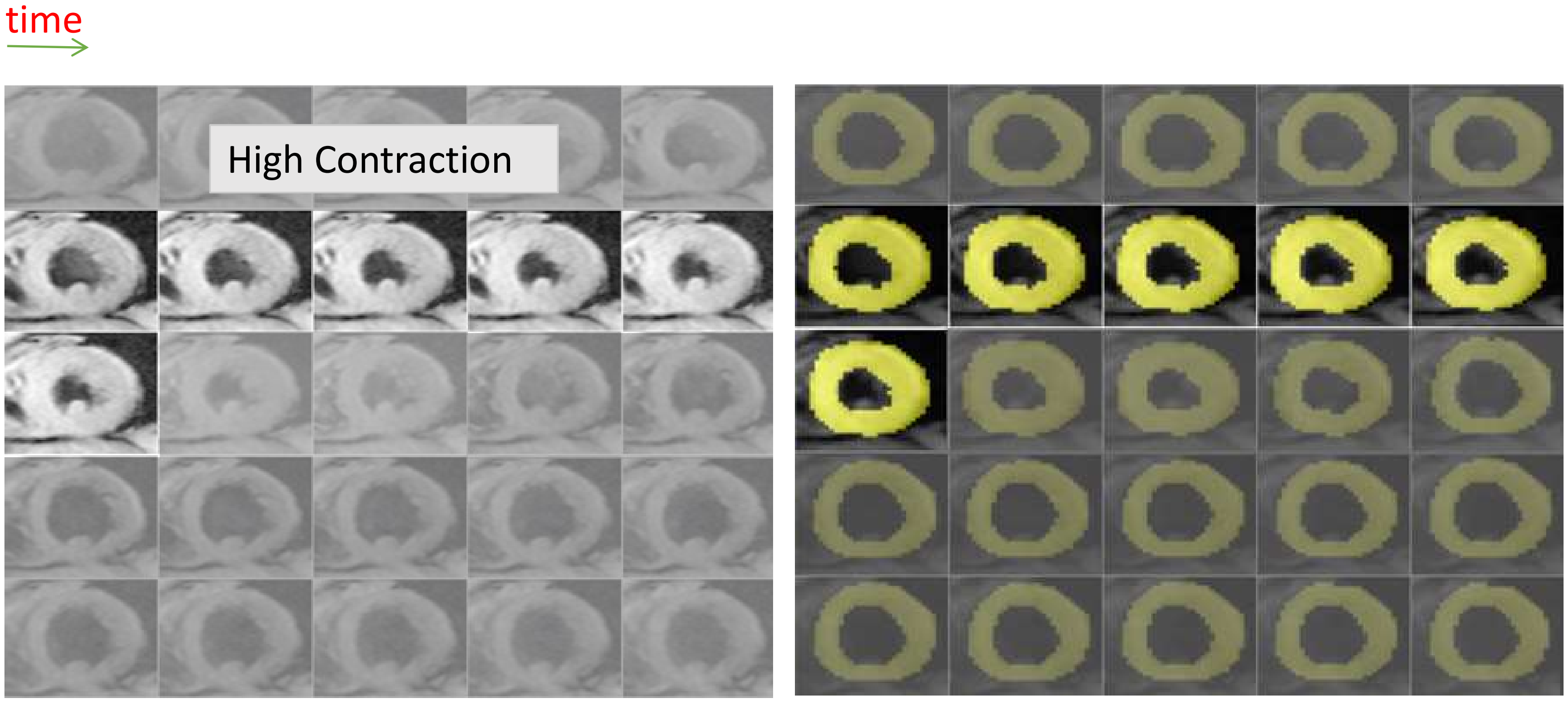
Proposed Method: Example

- 25 cardiac MRI images of one volunteer over 1 ECG cycle

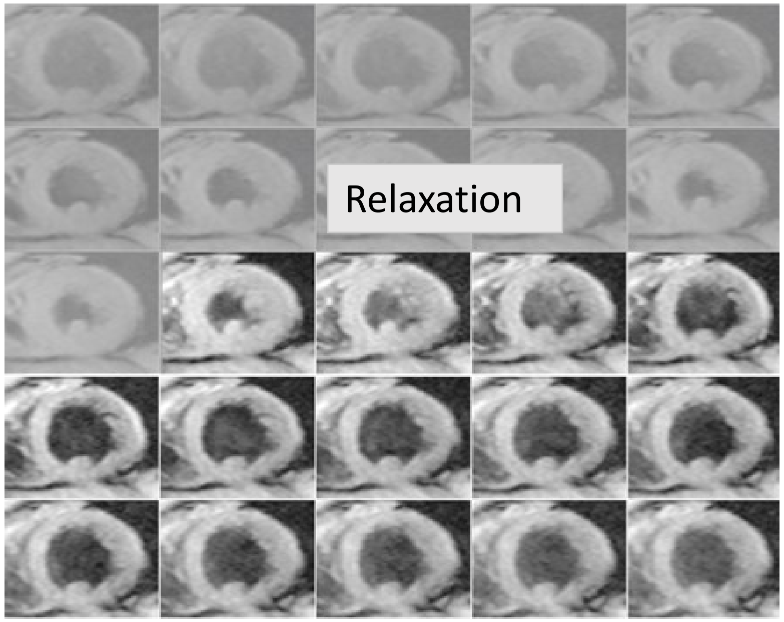


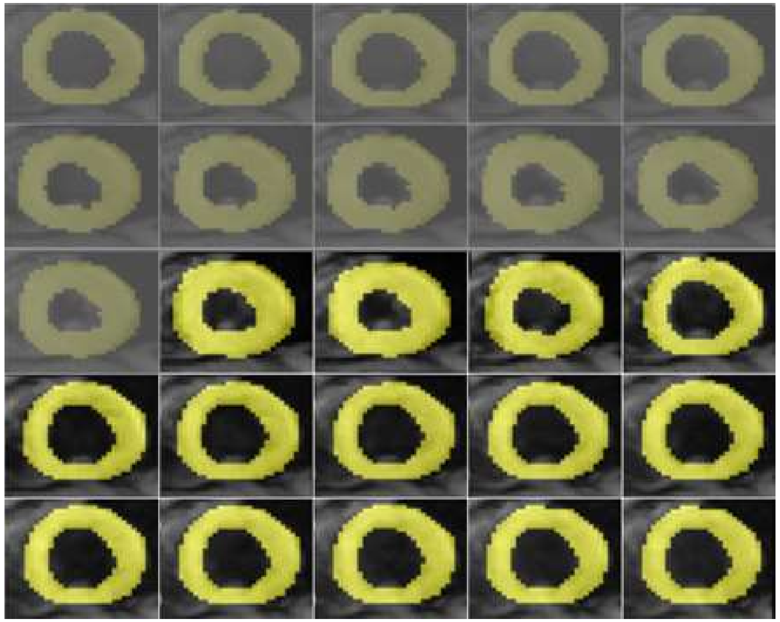
[E.Ruiz et.al, TIP, 2013]

Proposed Method: Example

- 25 cardiac MRI images of one volunteer over 1 ECG cycle

time

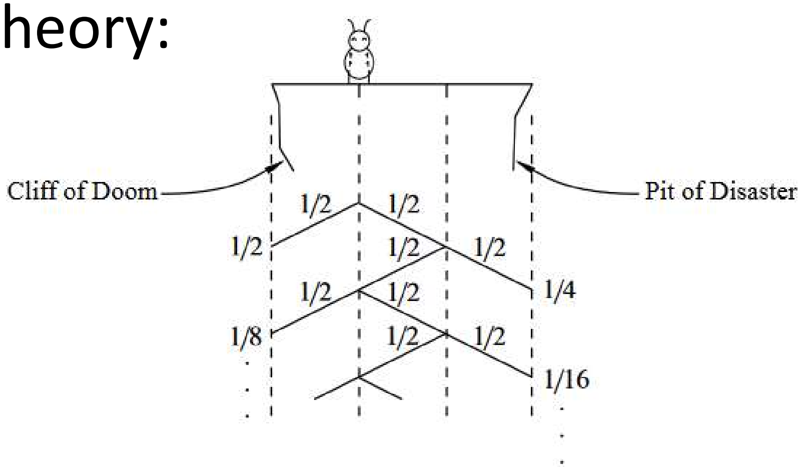




[E.Ruiz et.al, TIP, 2013]

Random Walks

- Random walks algorithm is a graph-based segmentation method. It is a good candidate due to its numerous advantages: [Leo Grady, PAMI, 2006]
  - Robust to noise
  - Robust to weak edges
  - Overcomes the drawbacks of graph cuts
- Theory:



he falls off the Cliff of Doom on the left side with probability:

$$\begin{aligned}\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\dots &= \frac{1}{2}\left(1+\frac{1}{4}+\frac{1}{16}+\dots\right)\\ &= \frac{1}{2}\cdot\frac{1}{1-1/4}\\ &= \frac{2}{3}.\end{aligned}$$

Pit of Disaster on the right side with probability:

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\dots = \frac{1}{3}.$$

[Source: mitopencourseware]

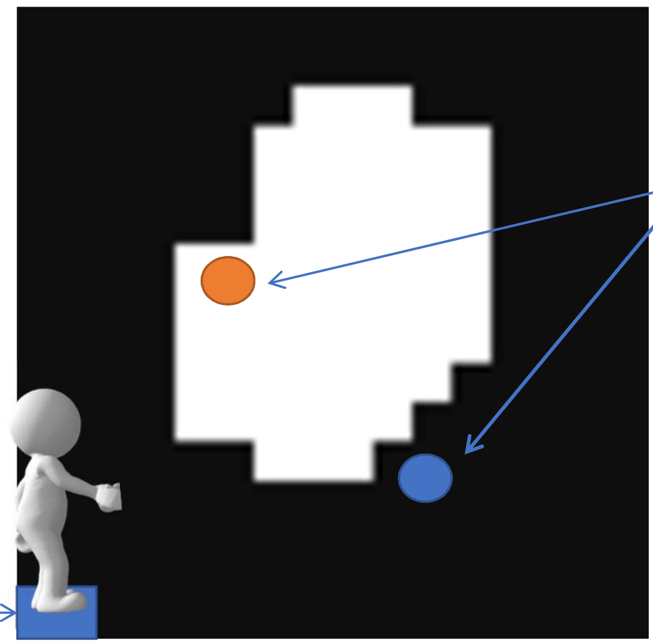
Random Walker

- Determine the probability that a random walker starting at each unlabeled pixel will first reach one of the seeds

Probability to reach the red seed

Probability to reach the blue seed

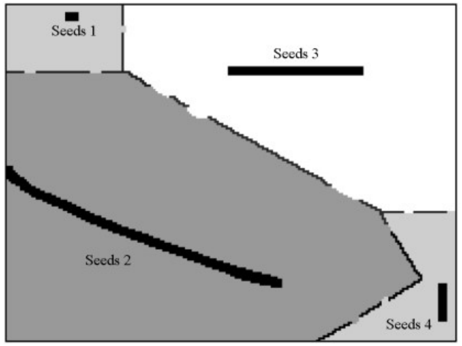
The label's seed with maximum probability is assigned to the unlabeled pixel



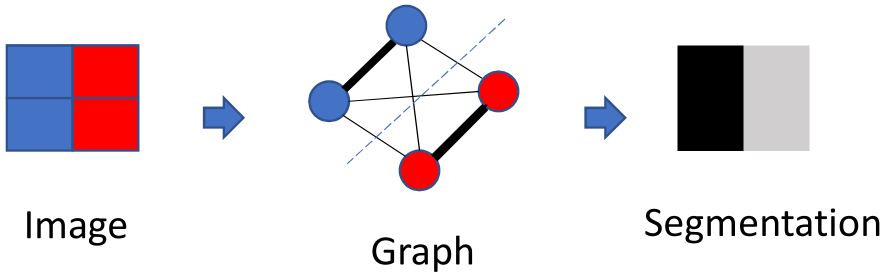
Seeds for background and foreground

Pixel *i,j*

Standard Random Walks

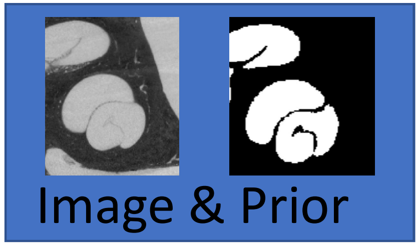


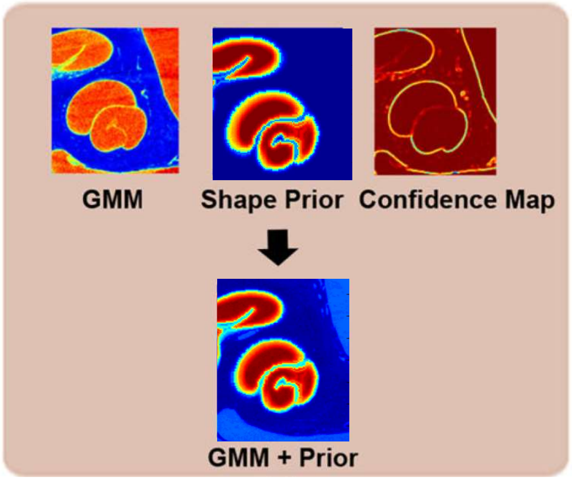
- Assign each pixel to the label for which it is most likely to reach first
- Image represented as a graph:
  - Pixels are nodes
  - Edges encode node similarity based on image features (weights)
- Small weights indicate less similarity and vice versa
- Partitions a graph according to the node similarity (biased by edges)



Extension of Random Walks

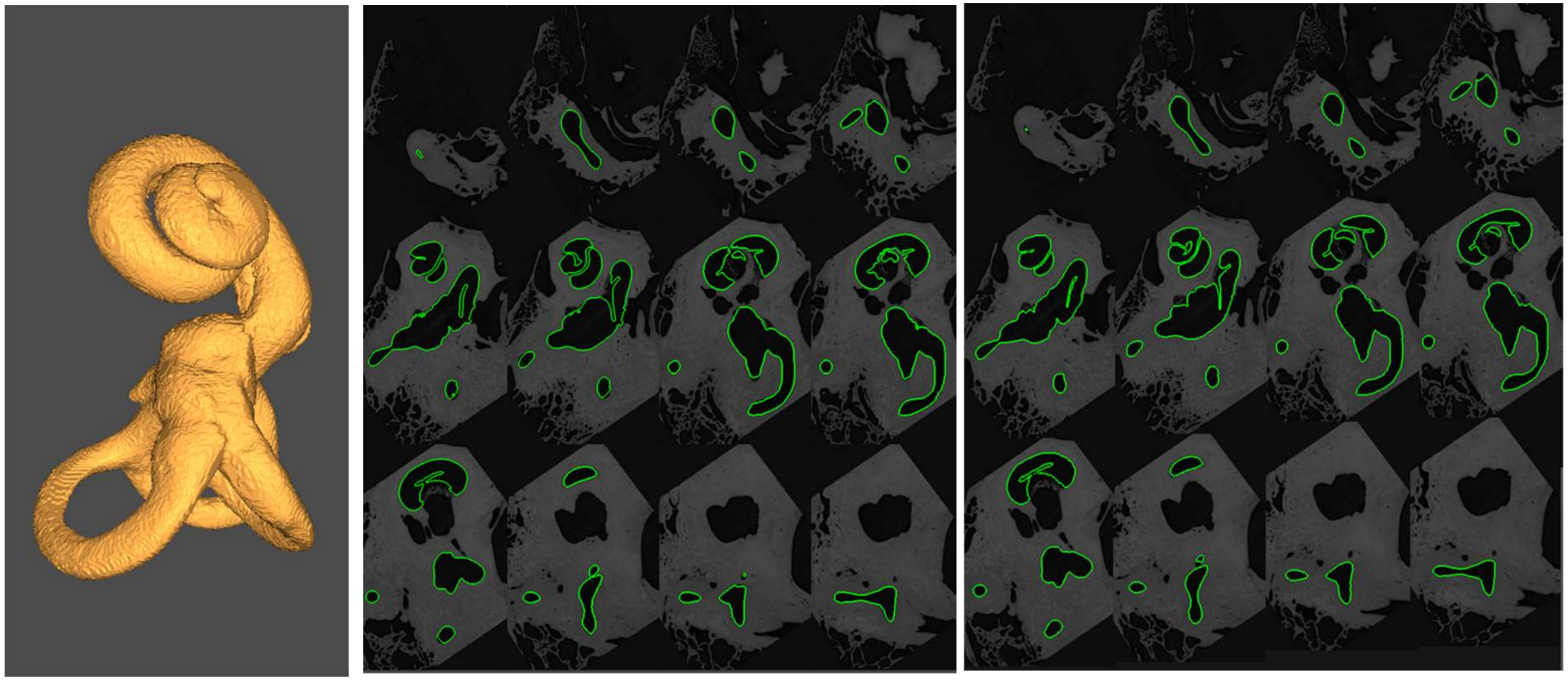
- No seeds are required unlike the standard random walks
- Probability distribution is introduced into the random walks [Leo Grady, CVPR, 2005]
- The new formulation allows a flexible way to introduce shape priors





[E.Ruiz et.al., Mach Vis Appl. 2015]

Proposed Method: Example



(a) (b) (c)

Inner Ear segmentation. (a) Segmentation in 3D. (b) Slices of the 3D segmentation.
(c) Ground truth

[E.Ruiz et.al., Mach Vis Appl. 2015]

3 – Coexistence with Deep Learning Methods

Deep Learning and Graph Search

Automatic segmentation of nine retinal layer boundaries in OCT images of non-exudative AMD patients using deep learning and graph search

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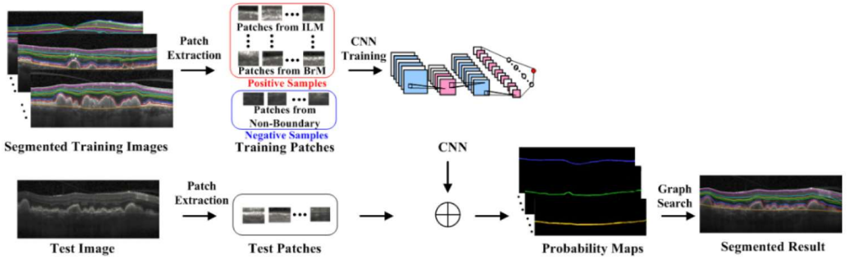
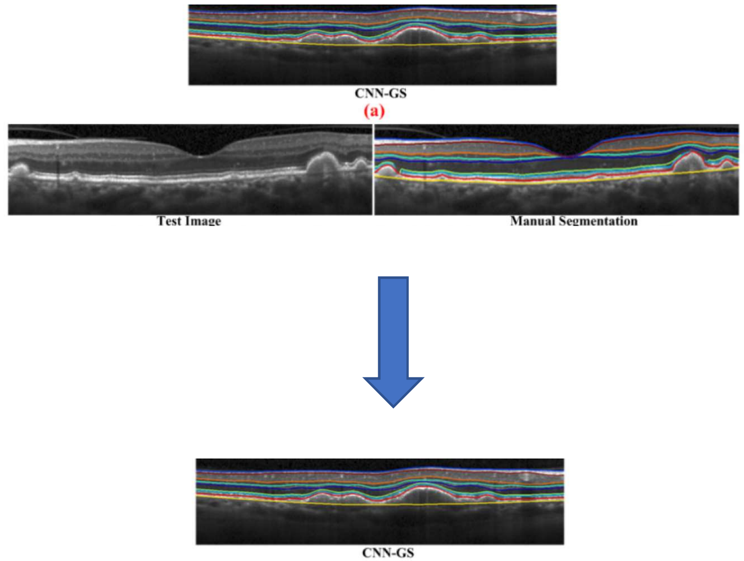


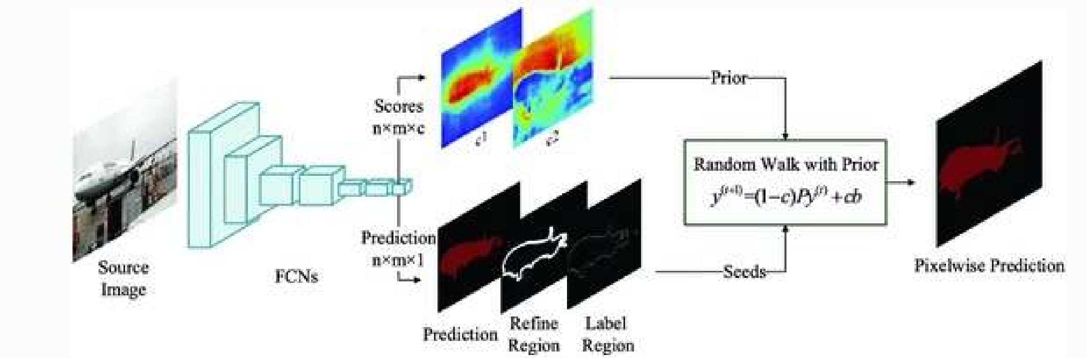


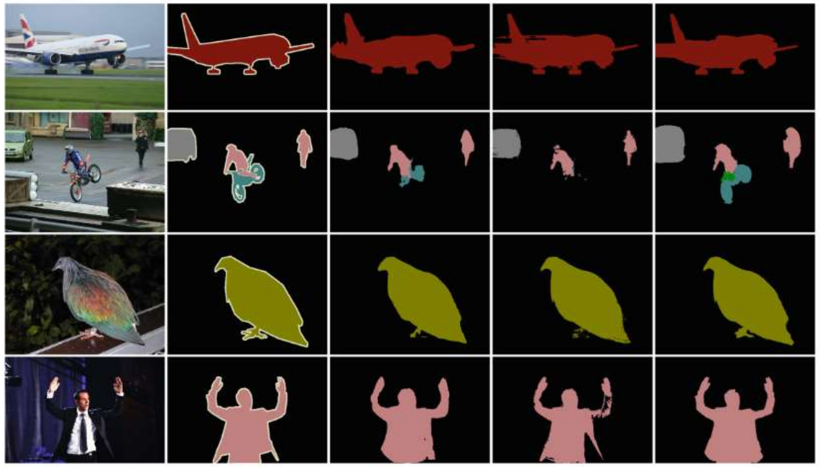
Deep Learning and Random Walks

Semantic Segmentation Using Fully Convolutional Networks and Random Walk with Prediction Prior

Xiaoyu Lei(✉), Yao Lu(✉), Tingxi Liu(✉), and Xiaoxue Shi(✉)

Beijing Laboratory of Intelligent Information Technology,
School of Computer Science, Beijing Institute of Technology,
Beijing 100081, China
{leixiaoyu,vis_yl,liutx,shixiaoxue}@bit.edu.cn





(a) Source (b) GT (c) Deeplab (d) Deeplab+CRF (e) Deeplab+RW

Graph Convolution Network

Attention Graph Convolution Network for Image Segmentation in Big SAR Imagery Data

Fei Ma [1], Fei Gao [1,*], Jinping Sun [1], Huiyu Zhou [2] and Amir Hussain [3]

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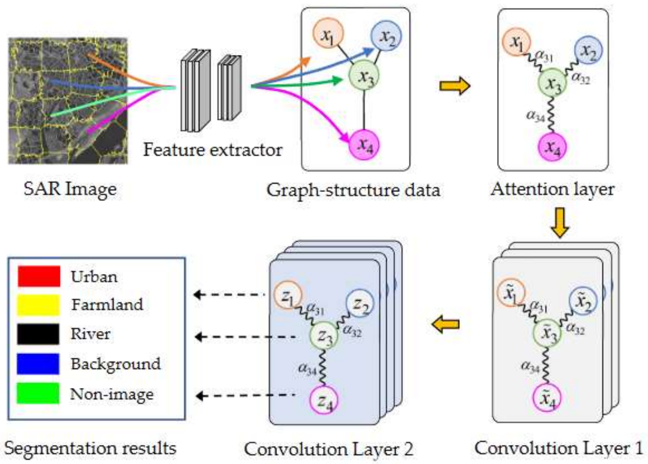
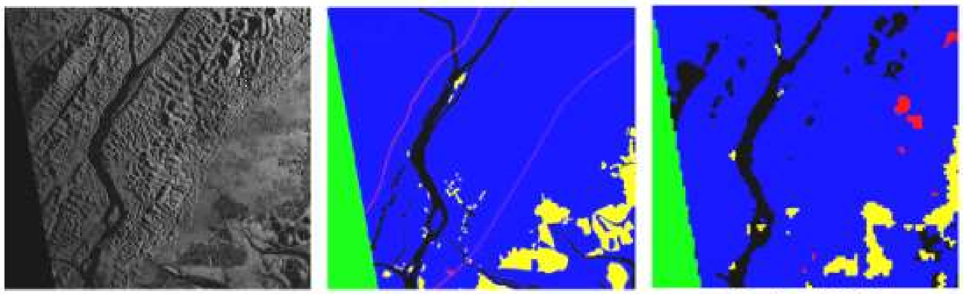


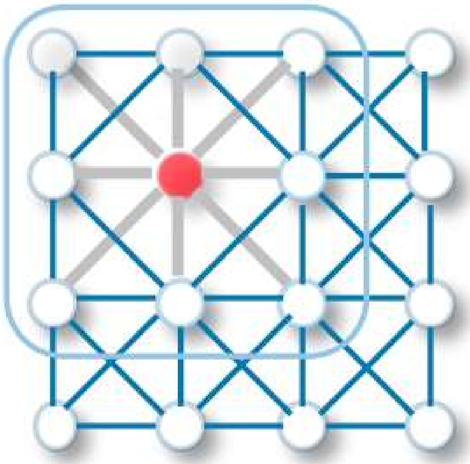


(a) Test image (b) Ground Truth (c) AGCN

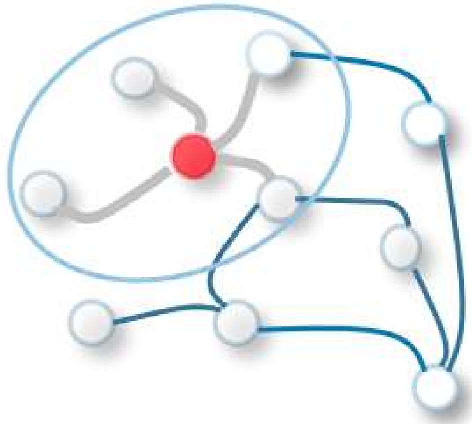
3 – Graph Convolutional Methods

Theory: Graph Convolution Network

- It can work on data with underlying non-regular structures
- The insertion of Adjacency Matrix (**A**) enable the model to learn the features of neighboring nodes



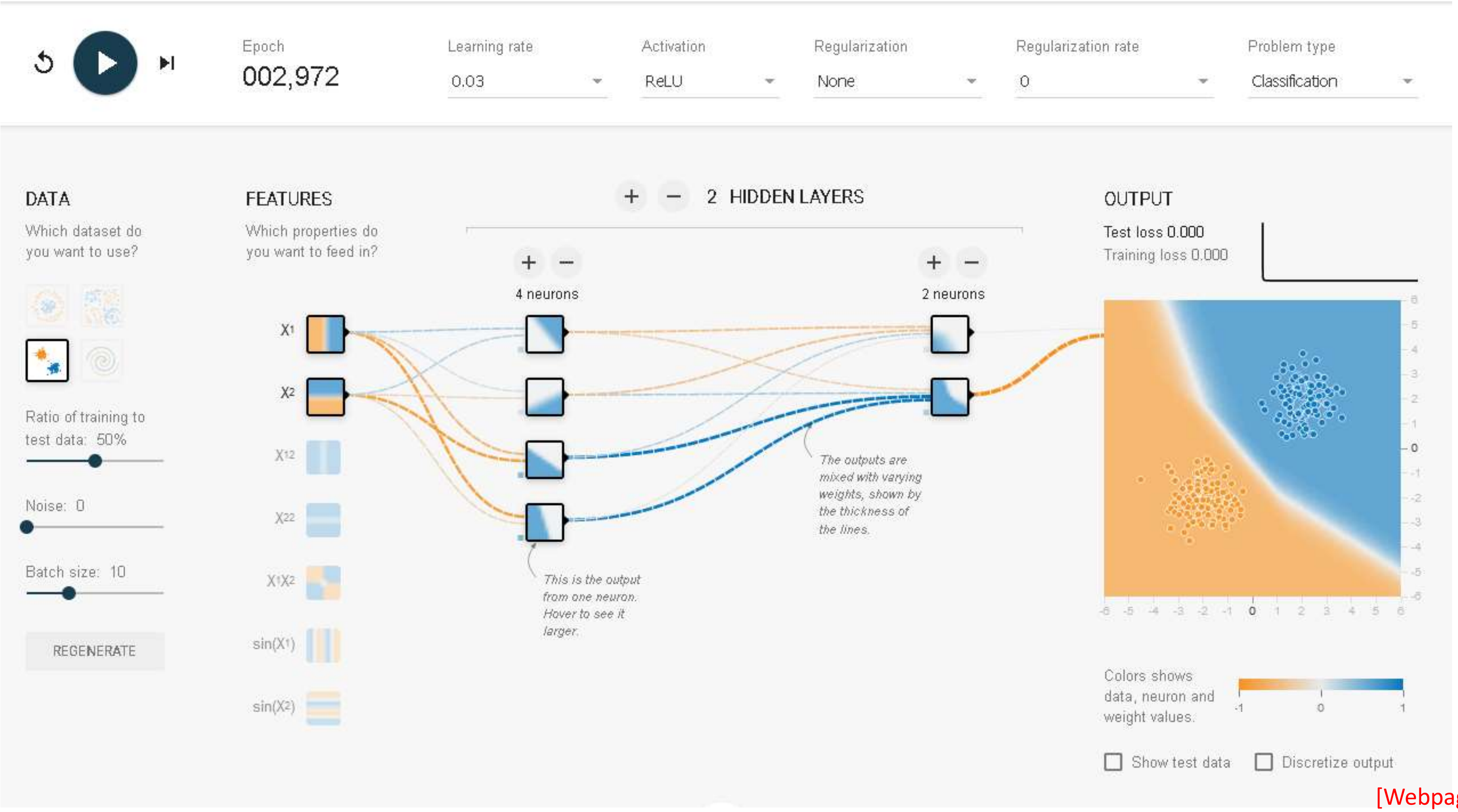
2D Convolutional Neural Networks



Graph Convolution Network

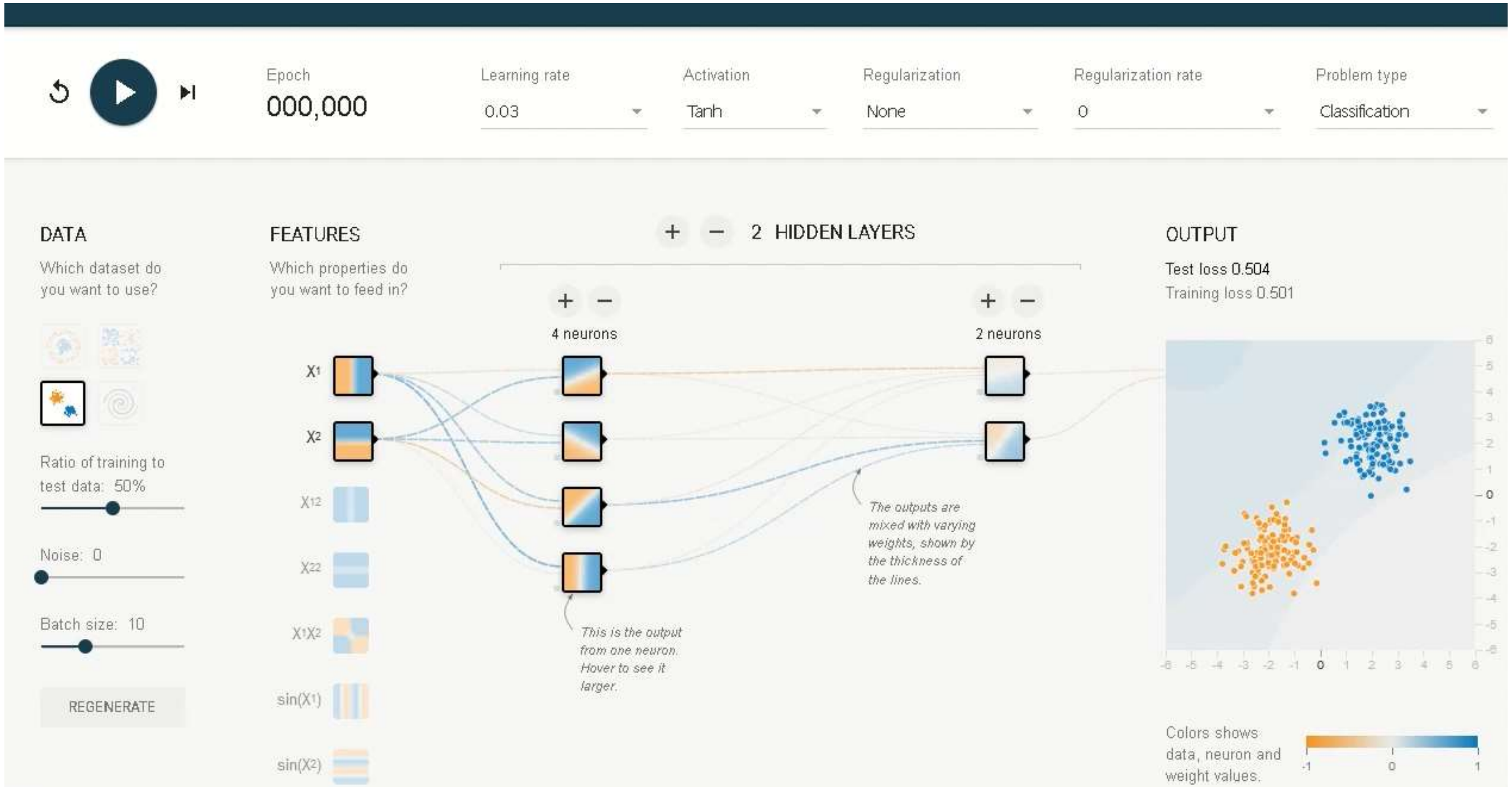
[Zonghan Wu et al., arXiv:1901.00596,2019]

Deep Learning Introduction



[Webpage: http://playground.tensorflow.org]

Deep Learning Introduction



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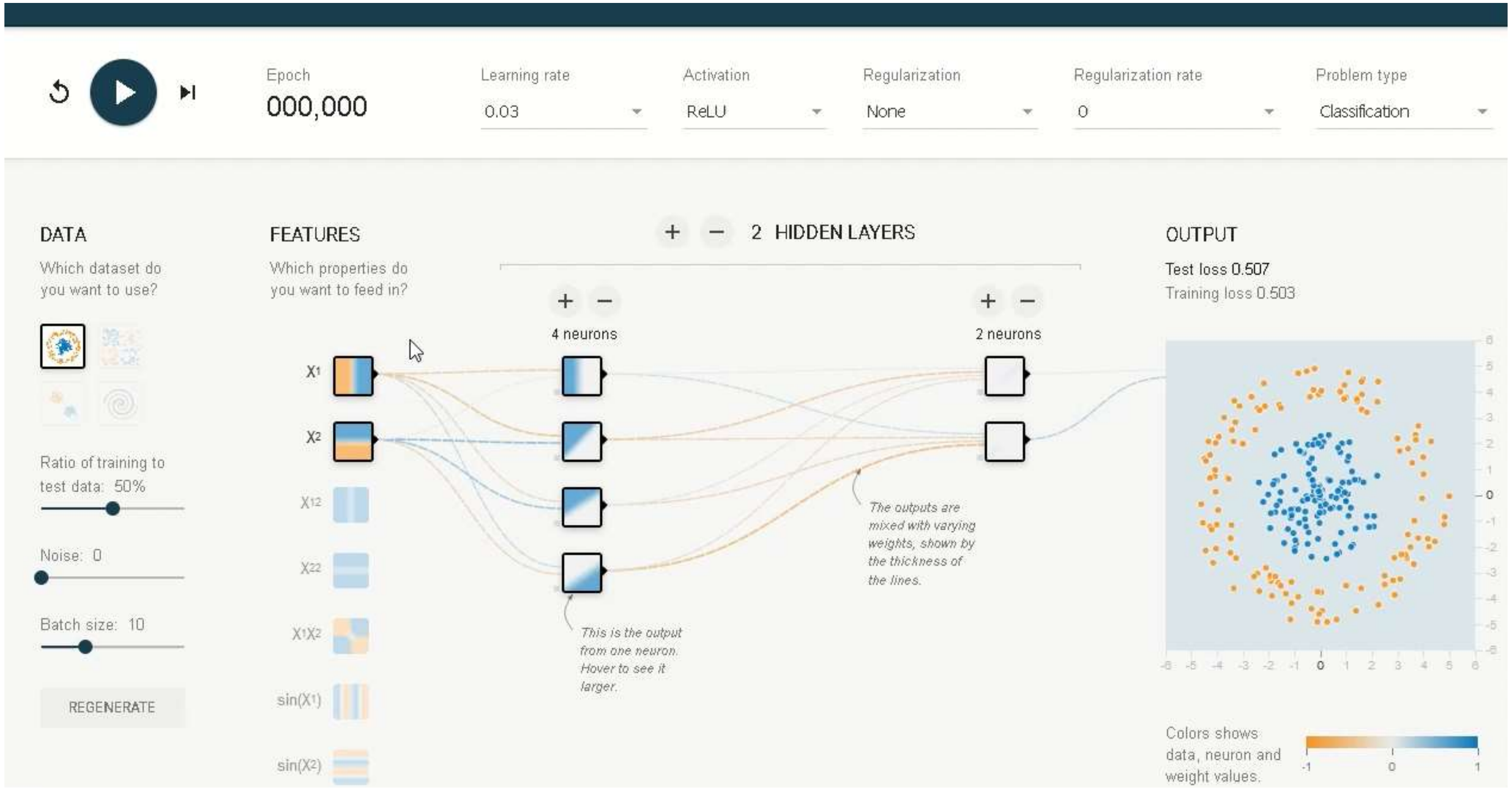
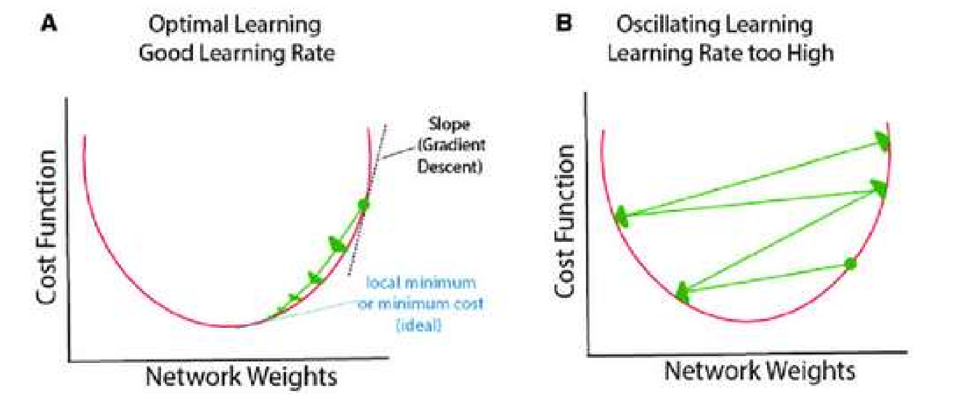
Deep Learning Introduction



[Webpage: http://playground.tensorflow.org]

Deep Learning Parameters

- **Learning rate:** controls how much change the model in response estimated error each time model weights are updated



- **Epochs:** number of times that the deep learning algorithm will work through the entire training dataset

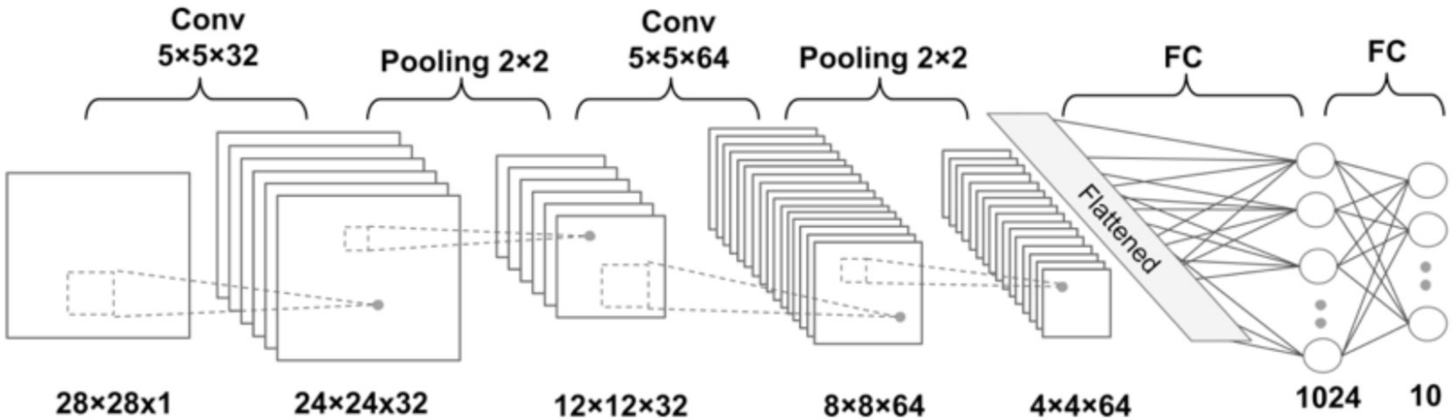
- **Bath size:** subdivide the training data into batches to pass it to the network

- **Activation function:** to propagate forward the output of the node

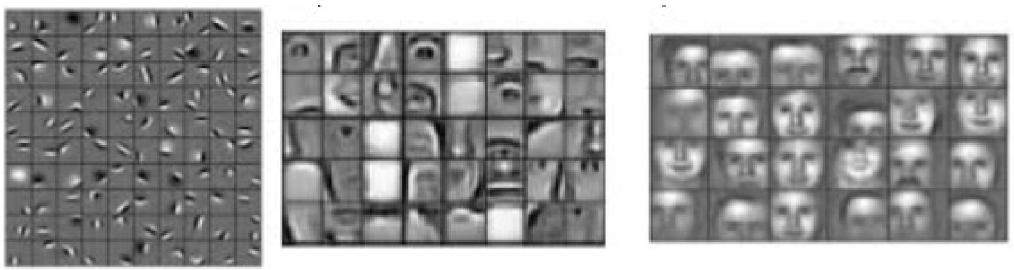
[Chayakrit Krittanawong et al.,Eur. Heart Journal, 2019]

Theory: Convolution Neural Networks

- Scheme



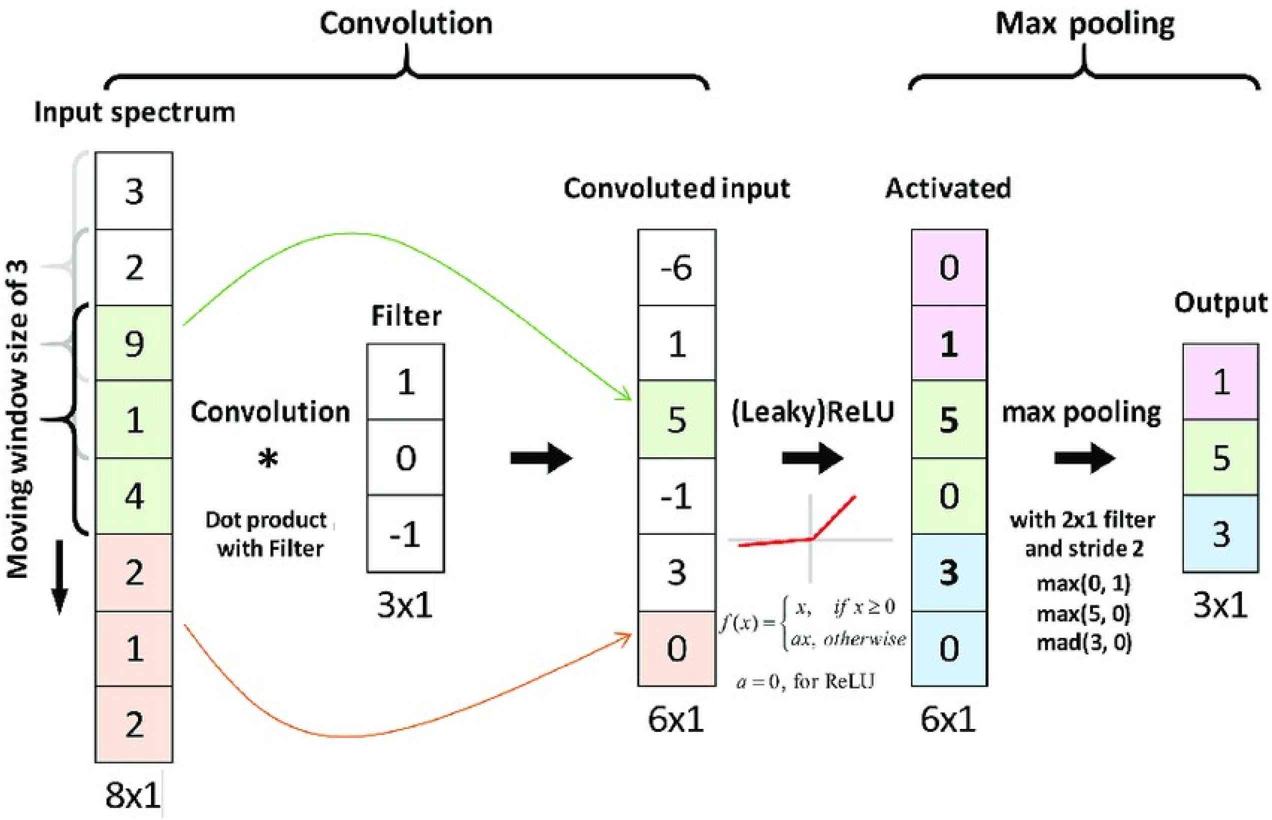
- Example of features



Low level Medium level High level

[Sebastian R. et al., Python ML,Packt Publishing Ltd.,2015]

Example of Convolutional Neural Networks



[Wooje Lee et al., J Raman Spectroscopy,2019]

Graph Convolution via Fast Approximate Spectral Graph

$$H^{[i+1]} = \sigma(W^{[i]} H^{[i]} + b^{[i]})$$

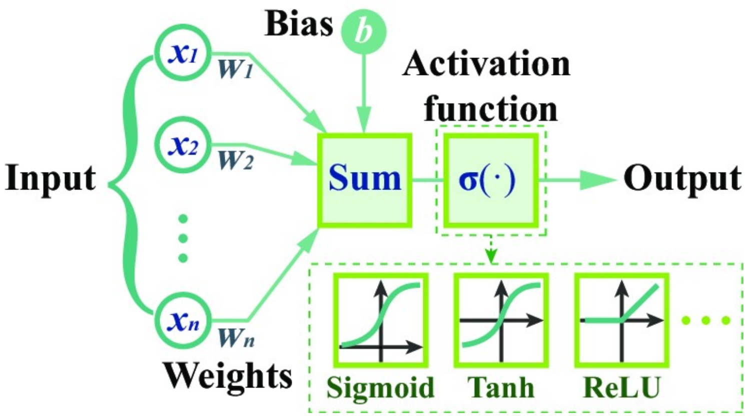
feature representation at layer i+1

activation function

weights at layer i

feature representation at layer i

bias at layer i



Forward Pass in Neural Networks

$$H^{[i+1]} = \sigma(W^{[i]} H^{[i]} A^{*})$$

Normalized Adjacency Matrix considering self-loop

$$H^{[i+1]} = \sigma(\tilde{D}^{\frac{-1}{2}} \tilde{A}\, \tilde{D}^{\frac{-1}{2}} H^{[i]}\, W^{[i]})$$

$$\tilde{A} = A+Id$$

[Inneke Mayachita, towardsdatascience,2020; Zhengjing Ma et al., Earth-Science Reviews ,2021]

3 – Conclusion and Discussion

Conclusion

- Graph-Based Segmentation Methods
  - A large number of samples is not required
  - The incorporation of priors requires to change the formulation
  - The representation of the image is sparse
- Deep Learning Methods
  - A big number of datasets is required
  - The use of deep learning methods is straightforward
- Hybrid
  - Combination of information obtained by graphs and deep learning techniques
  - It can learn features representation even before training

Thank you for your attention