A computational theory for the production of limb movements

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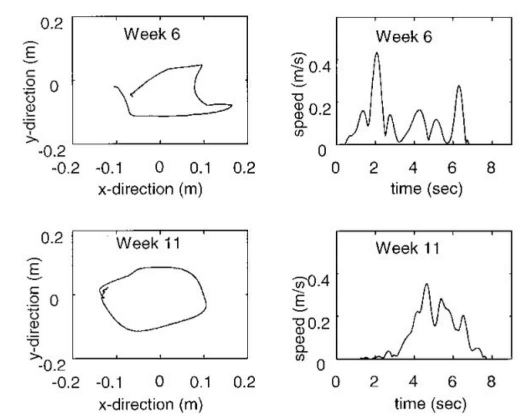
UNDERSTANDING MOTOR CONTROL

"To move things is all that Mankind can do … For such the sole executant is muscle, whether in whispering a syllable or in felling a forest." — Charles Sherrington, 1924, *The Linacre Lecture*

- **Action reflects decision, motivation, emotion, …**
— as important as reaction times and error rates

- **Pathological movements**
— stroke, Parkinson's disease, cerebellar disorders: rehabilitation, substitution, augmentation



MIT–Manus

— Krebs et al., 1999, *Proc Natl Acad Sci USA* 96:4645

- **Inspiring human-like robotics**

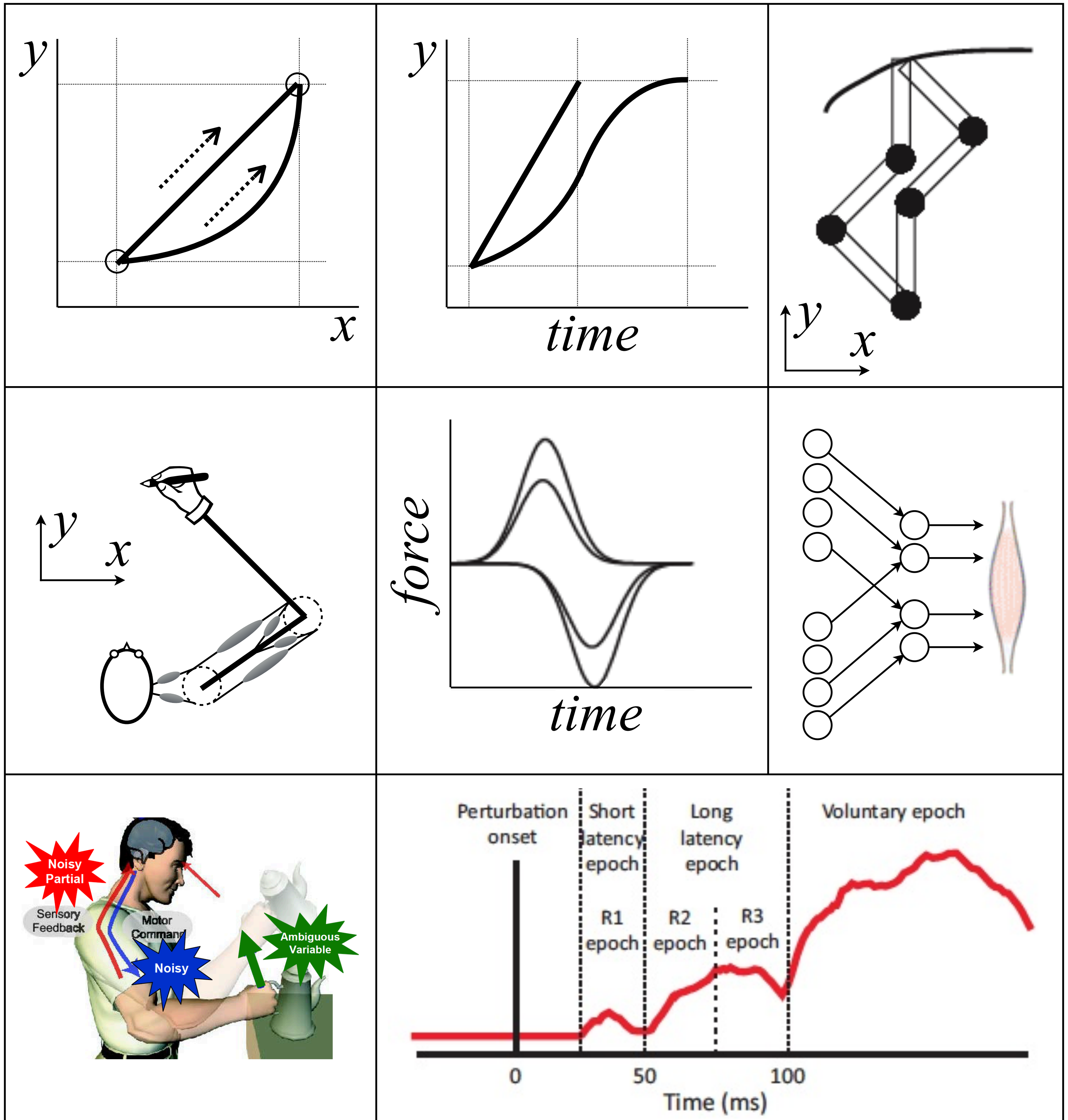
Miroki robot (*Enchanted Tools*)

PROBLEMS

redudancy

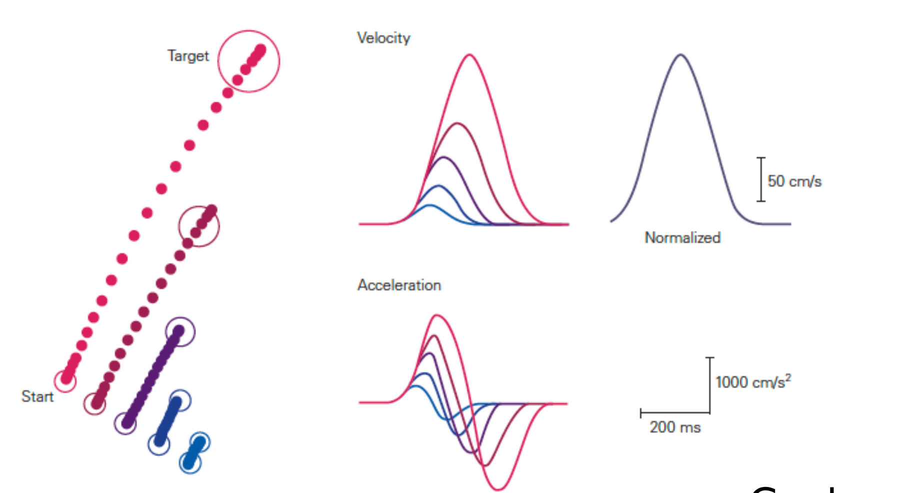
noise

delay

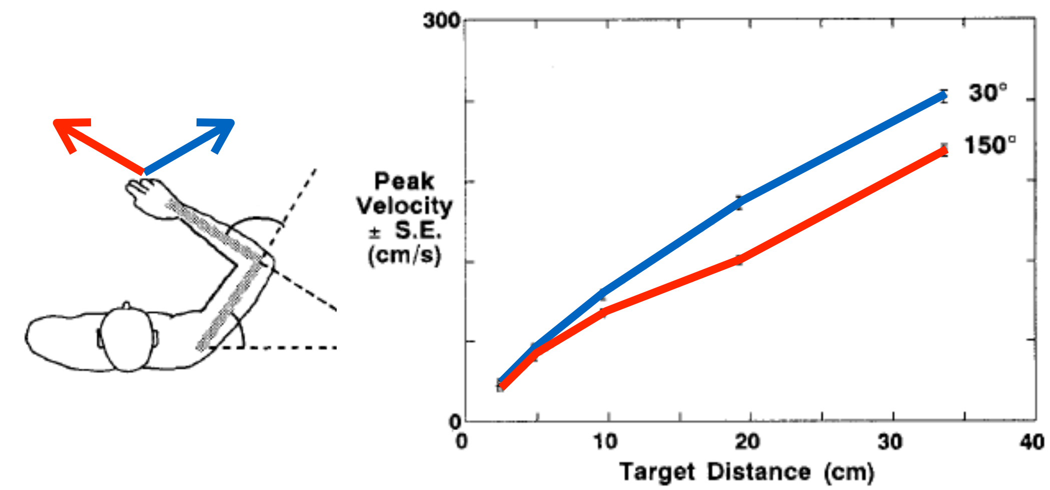


GENERAL PROPERTIES

invariant

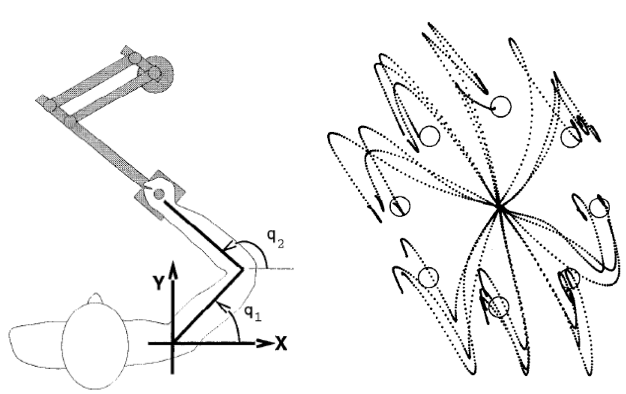


scaling

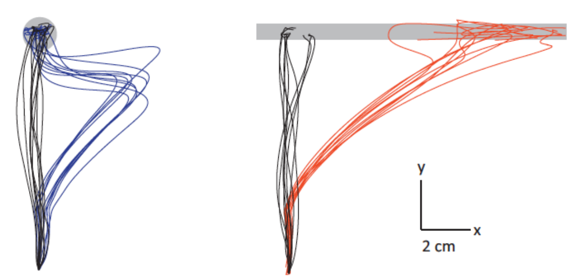


— Gordon et al., 1994, *Exp Brain Res* 99:112

flexibility

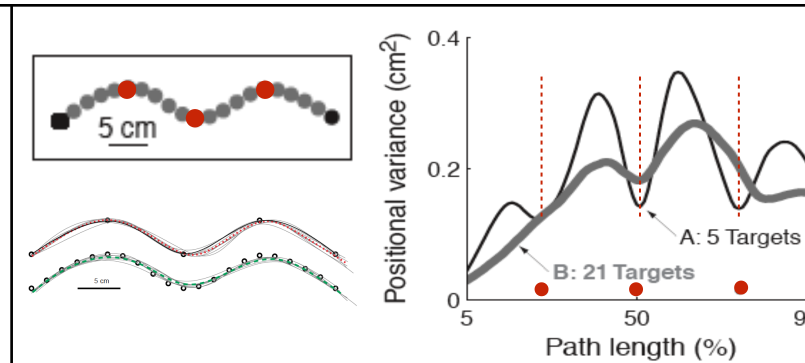


— Shadmehr & Mussa-Ivaldi, 1994, *J Neurosci* 14:3208



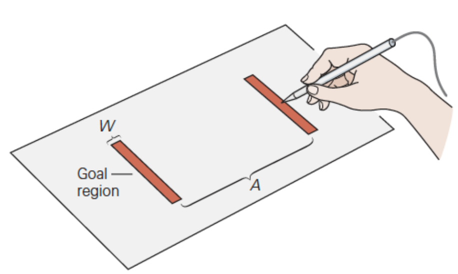
— Nashed et al., 2012, *J Neurophysiol* 109:999

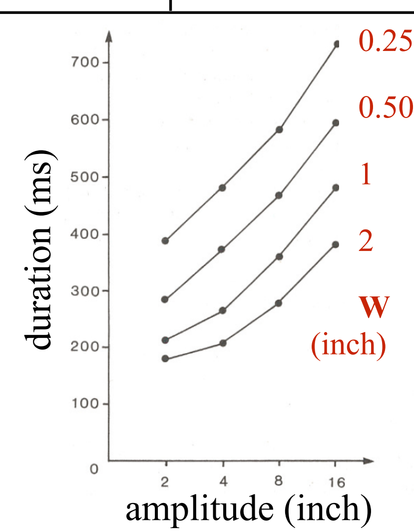
variability



— Todorov & Jordan, 2002, *Nat Neurosci* 5:1226

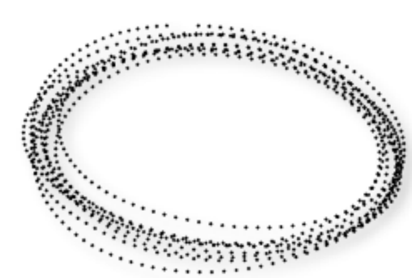
Fitts' law

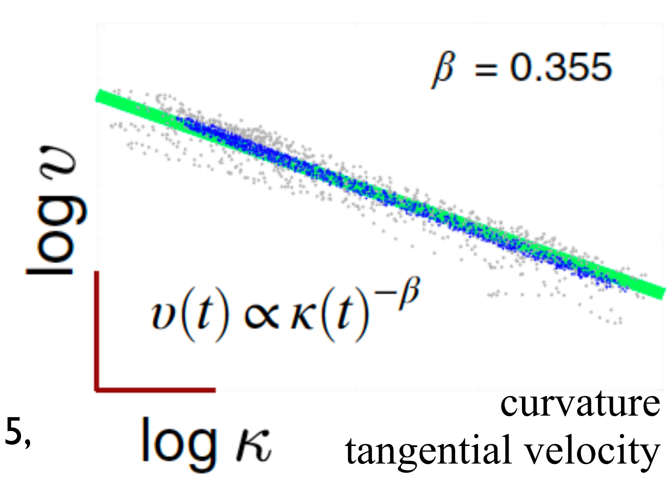




— Fitts, 1954, *J Exp Psychol* 47:381

power laws

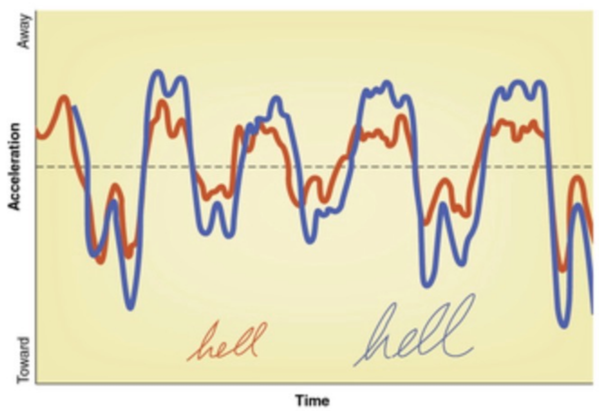




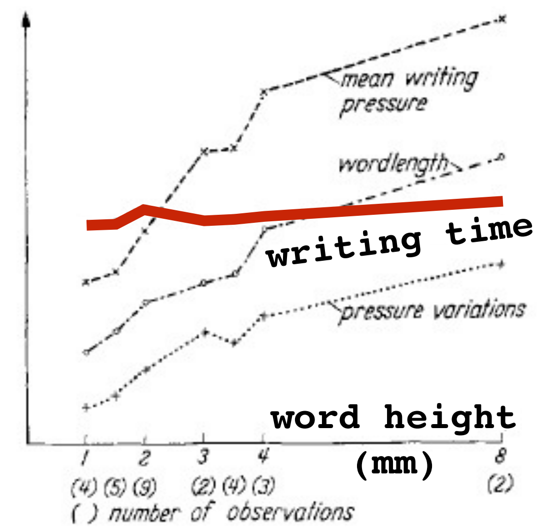
— Huh & Sejnowski, 2015, *PNAS* 112:E3950

SPECIFIC PROPERTIES

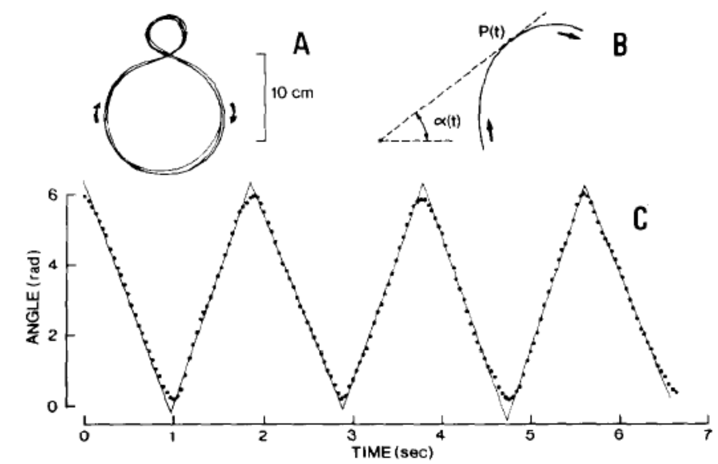
isochrony



— Hollerbach, 1978, *Doctoral Dissertation*, MIT

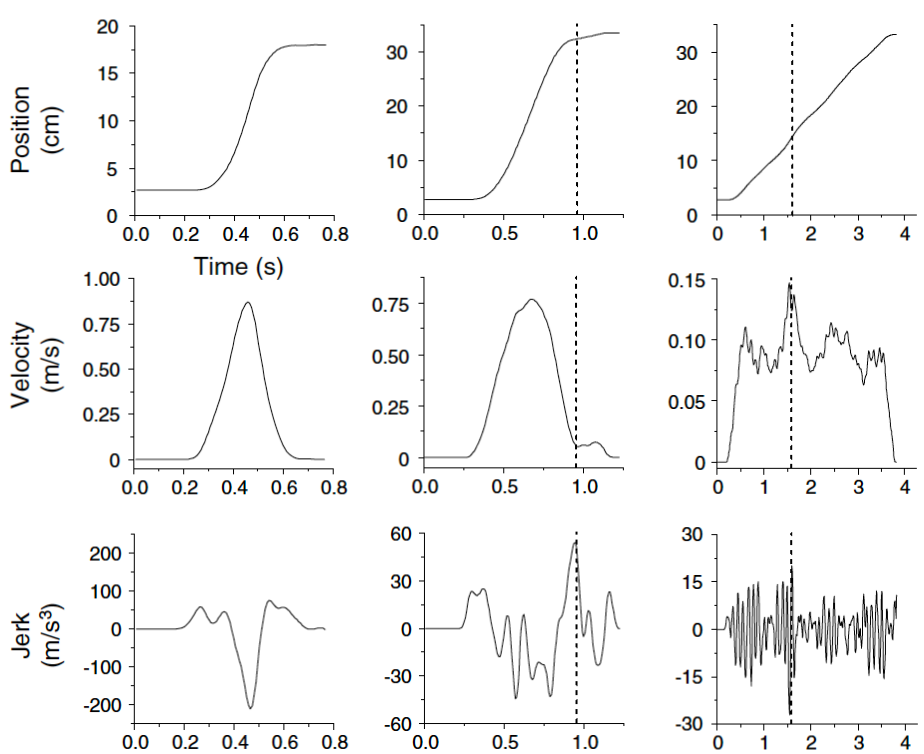


— Denier van der Gon & Thuring, 1965, *Kybernetik* 2:145



— Lacquaniti et al., 1983, *Acta Psychol* 54:115

segmentation

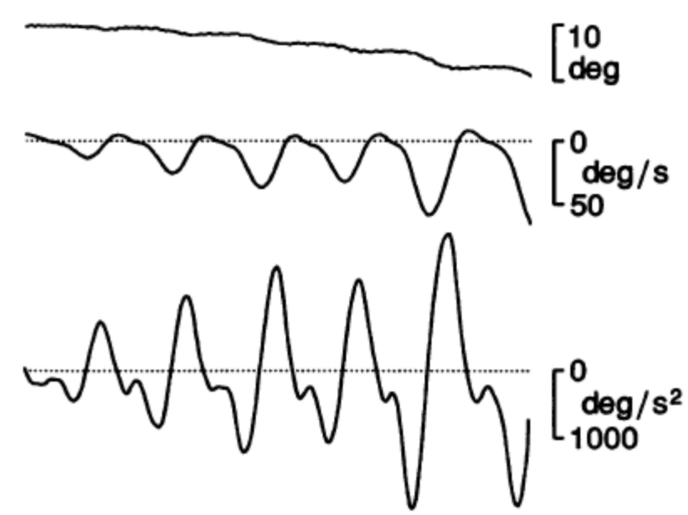


short distance, large target, maximum speed

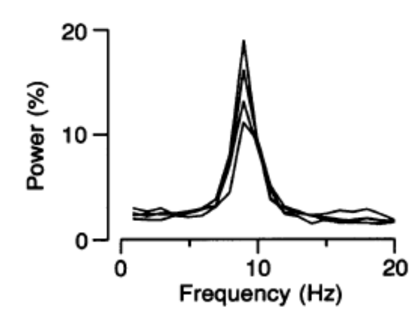
long distance, small target, maximum speed

long distance, small target, low speed

— Vallbo & Wessberg, 1993, *J Physiol (Lond)* 469:673



— Rand & Shimansky, 2013, *Exp Brain Res* 230:1



MODELING MOTOR CONTROL

- **« Task dynamics »**

— generalized closed-loop systems
— movements results from convergence to attractors of a dynamical system

Action systems approach
Dynamical systems
Ecological psychology

$$\dot{x}(t) = f(x(t), \{P\})$$

theory of dynamical systems

- **« Internal model »**

— builds an internal model of the system to follow a prescribed trajectory or match some constraints (e.g. optimization)

Information processing
Cognitive approach
Motor programs

$$\dot{x}(t) = f(x(t), u(t))$$

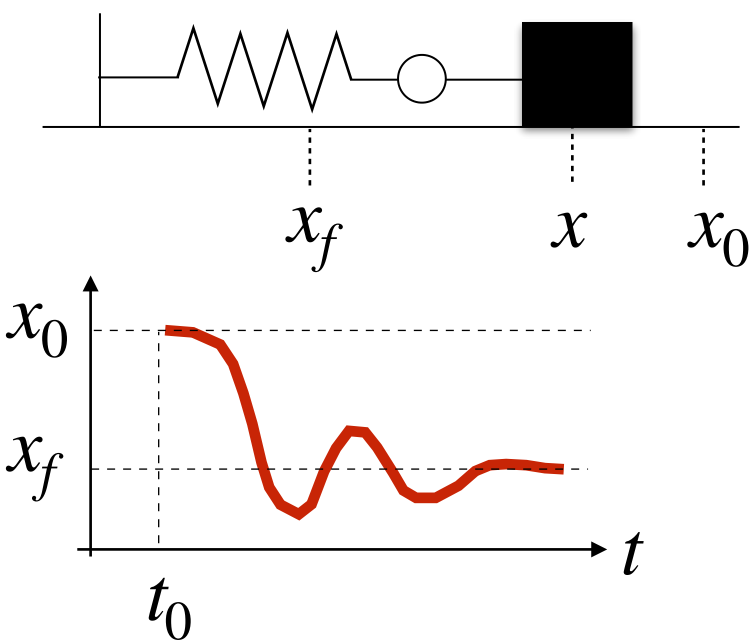
theory of control

MODELING MOTOR CONTROL

- « **Task dynamics** »

$$\mathrm{m}\ddot{x} + \mathrm{b}\dot{x} + \mathrm{k}(x - x_f) = 0$$

$$x = x(\mathrm{m}, \mathrm{b}, \mathrm{k}, t)$$

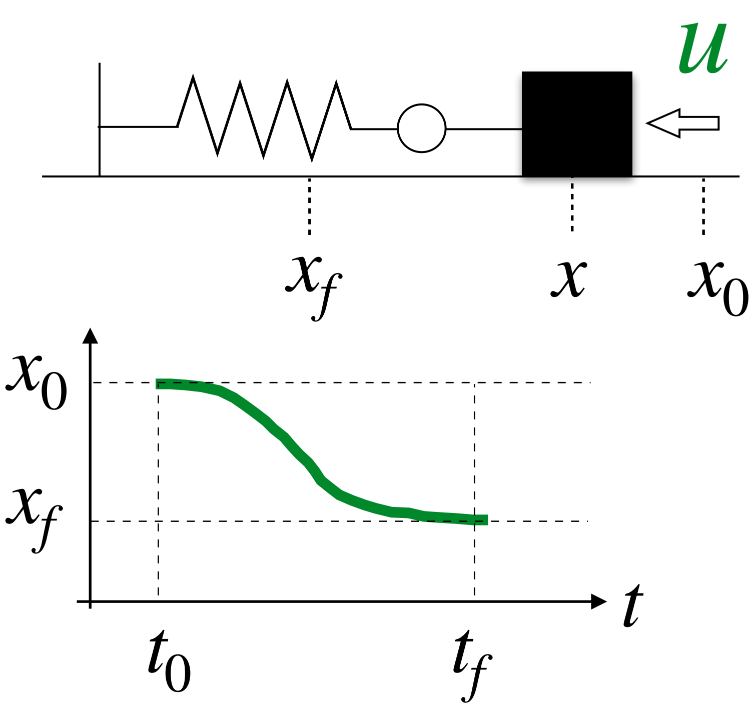


- « **Internal model** »

$$\mathrm{M}\ddot{x} + \mathrm{B}\dot{x} + \mathrm{K}(x - x_f) = u$$

$$u = u(\mathrm{M}, \mathrm{B}, \mathrm{K}, t)$$

$$x = x(t, u)$$

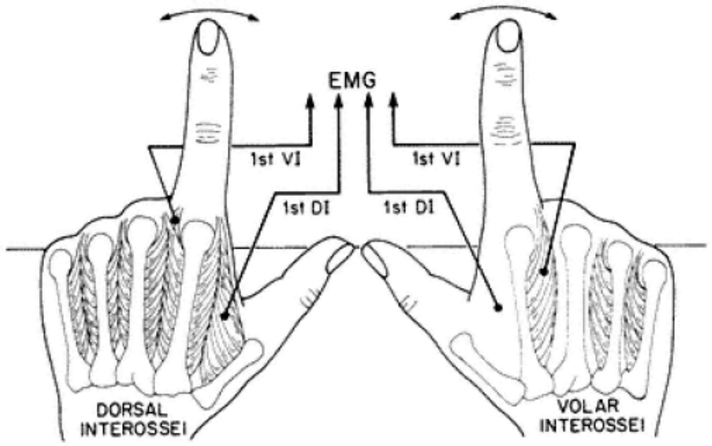


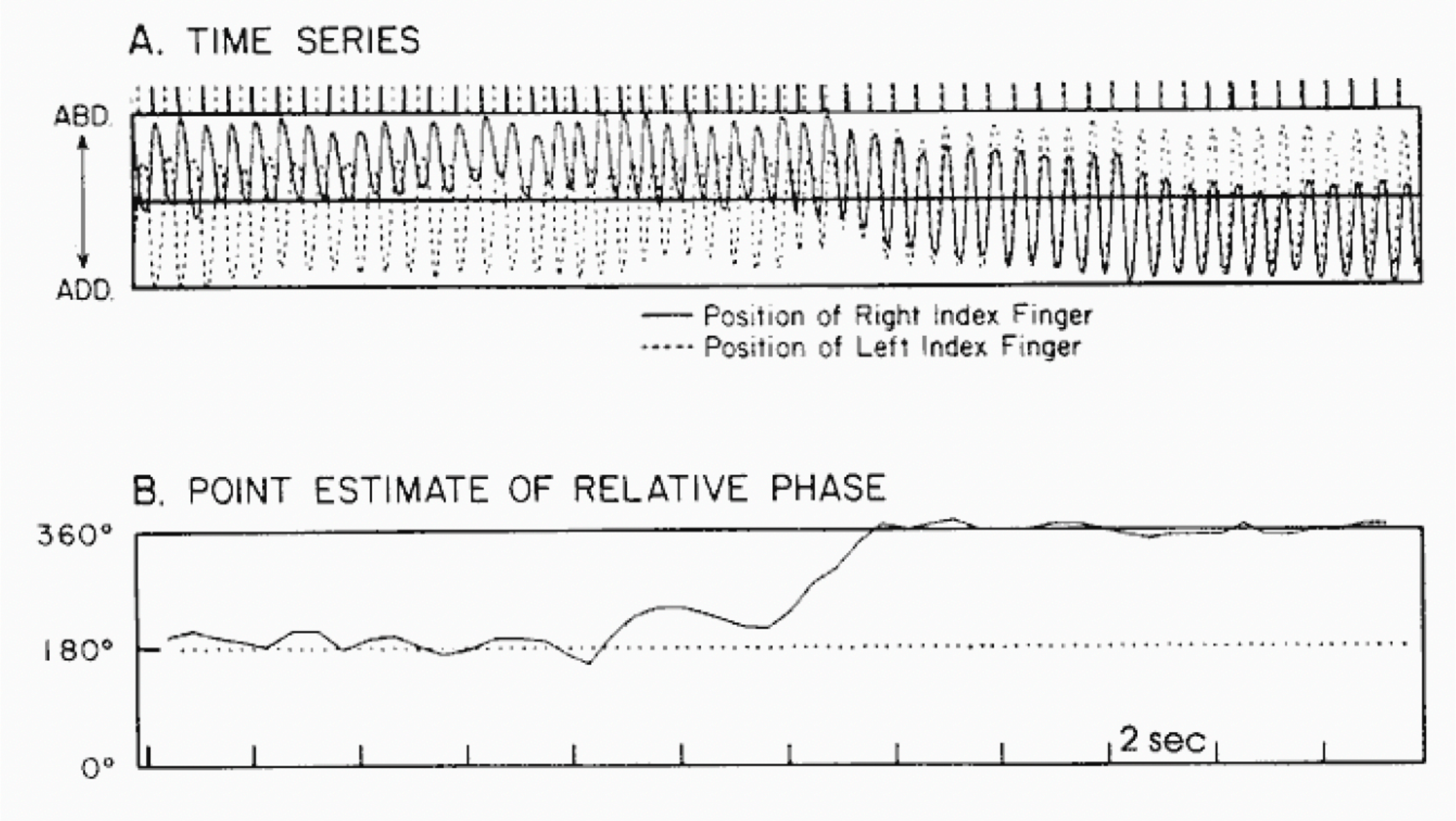
TASK DYNAMICS

Bimanual coordination

— start antiphase

— increasing frequency (1-5 Hz)





— Kelso, 1984, *Am J Physiol* 246:R1000

TASK DYNAMICS

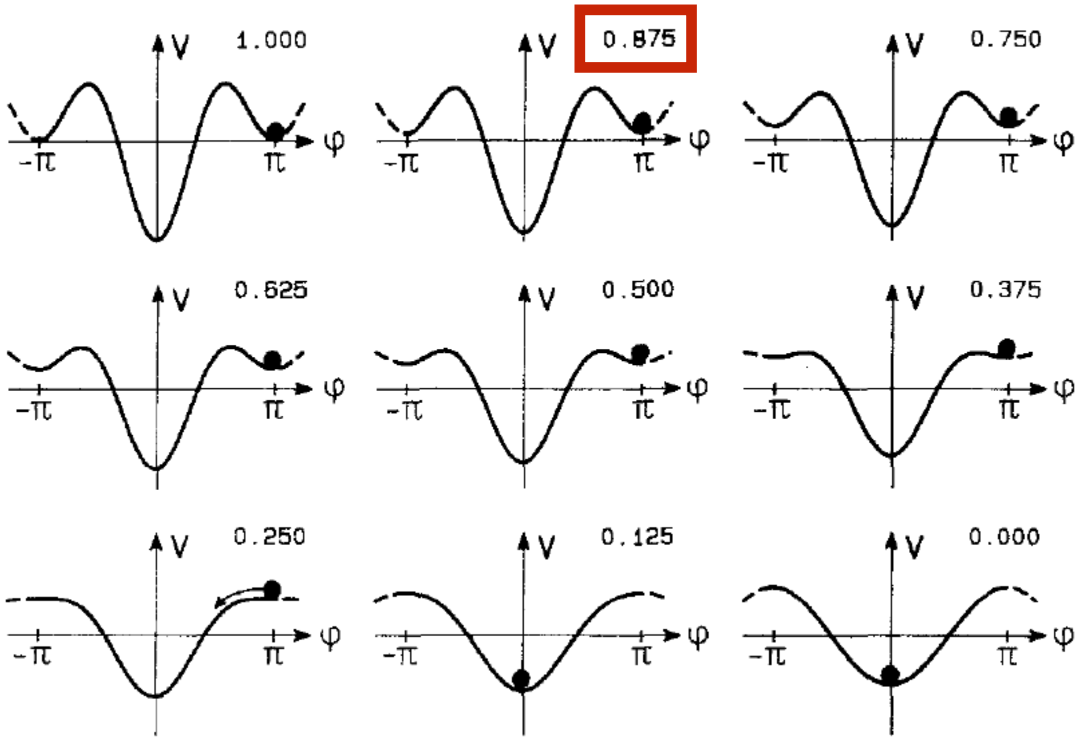
Bimanual coordination

— phenomenological model

$$\dot{\phi} = -\frac{dV}{dt}$$

$$V = -a\cos\phi - b\cos 2\phi$$

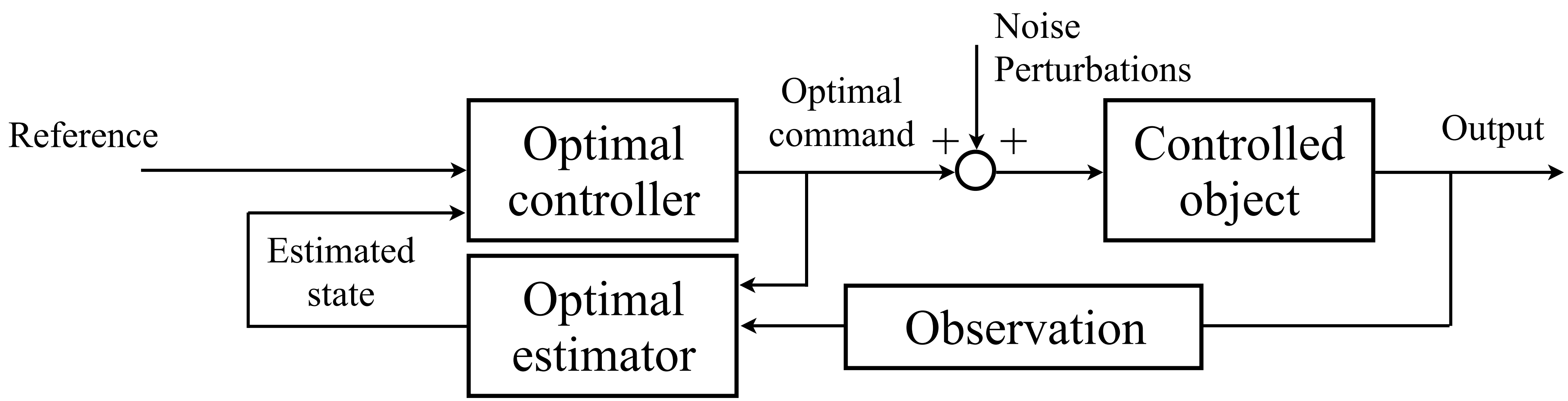
$\frac{b}{a} \downarrow \Rightarrow$ frequency $\uparrow$



— Haken et al., 1985, *Biol Cybern* 51:347

INTERNAL MODEL

- **Optimal feedback control**



— Todorov & Jordan, 2002, *Nat Neurosci* 5:1226

- **e.g. LQG (Linear Quadratic Gaussian)**

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{w}(t)$$

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{v}(t)$$

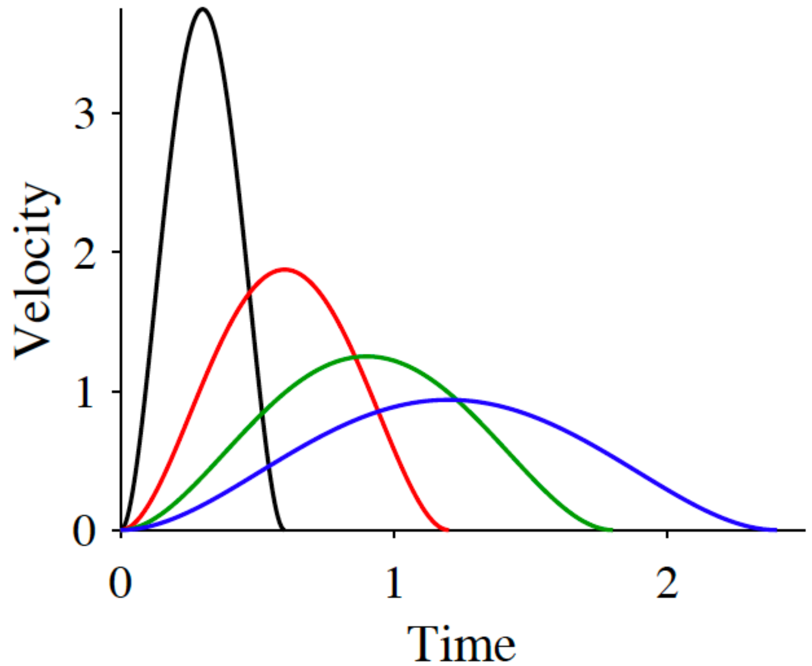
$$J = E\left(\int_{t}^{t_f} \left[\mathbf{x}^T(\tau)\mathbf{Q}(\tau)\mathbf{x}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\mathbf{u}(\tau)\right] \, d\tau \right), \quad t \in [t_0, t_f]$$

$$\dot{\hat{\mathbf{x}}}(t) = \underbrace{\mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t)}_{\text{forward model}} + \underbrace{\mathbf{K}(t)}_{\text{Kalman gain}}\Big(\underbrace{\mathbf{y}(t) - \mathbf{H}\hat{\mathbf{x}}(t)}_{\text{sensory prediction error}}\Big)$$

$$\mathbf{u}(t) = -\mathbf{L}(t)\hat{\mathbf{x}}(t)$$

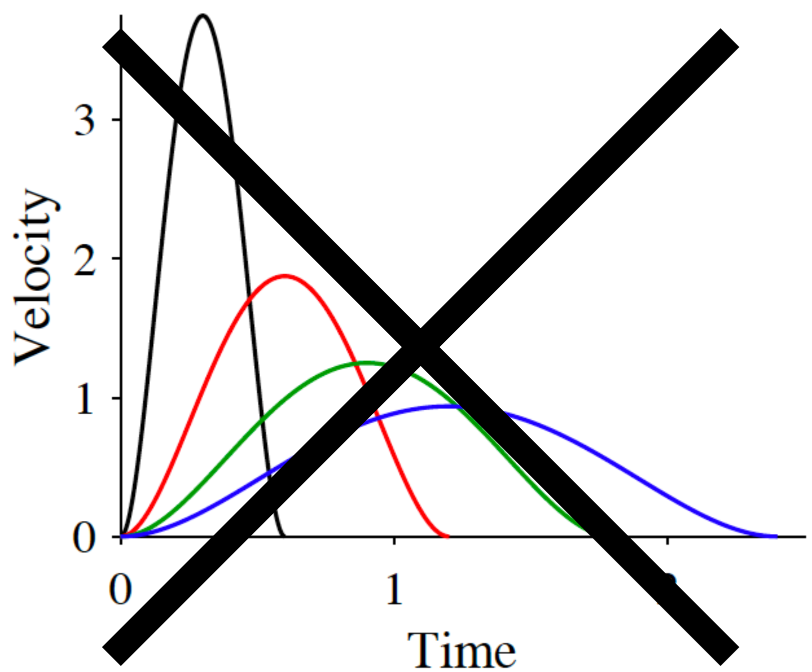
TIME IN MOTOR CONTROL

smoothness

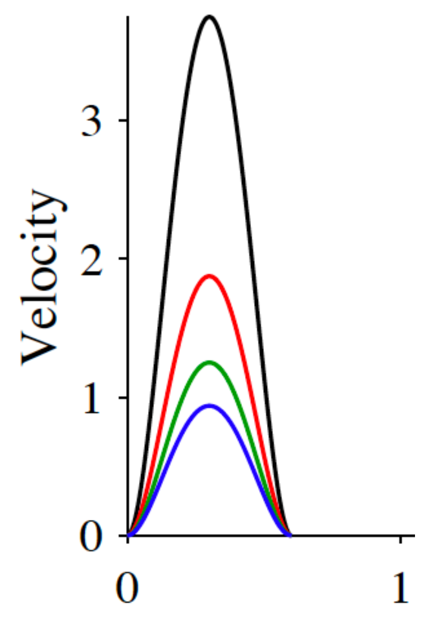


TIME IN MOTOR CONTROL

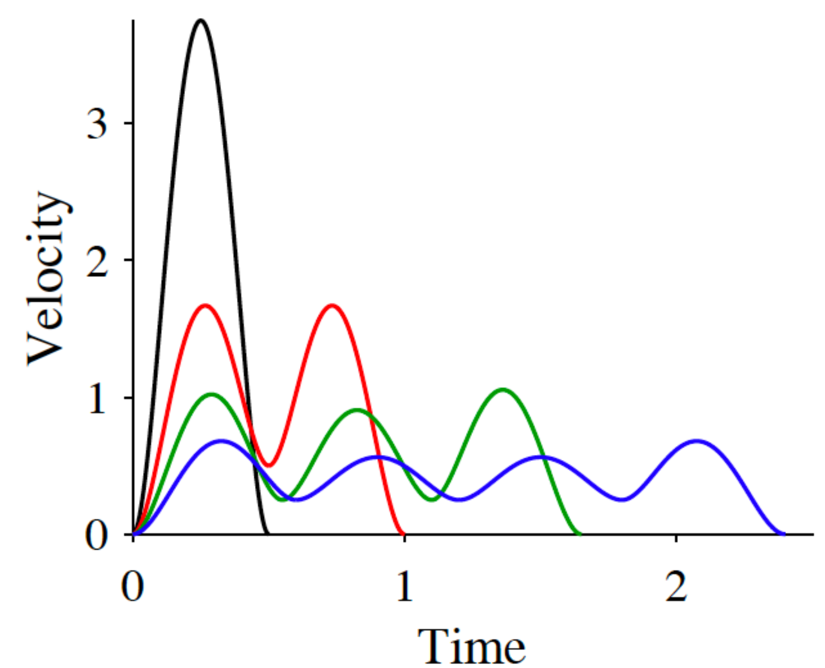
smoothness



isochrony



segmentation



MODELING PRINCIPLES

- **Motor control**

— **control problem** in which the behavior of a **controlled object** is governed by a **controller** through a **control policy** and a **series of goals** to achieve

- **Control policy**

— « universal » optimal feedback control policy

At each time
`current state`
`current goal`
`time to reach the goal`
⇩
`next control`

- **Time to reach a goal**

— constant irrespective of the time already spent for this goal (***receding Horizon***) ⇨ **stationary control policy**

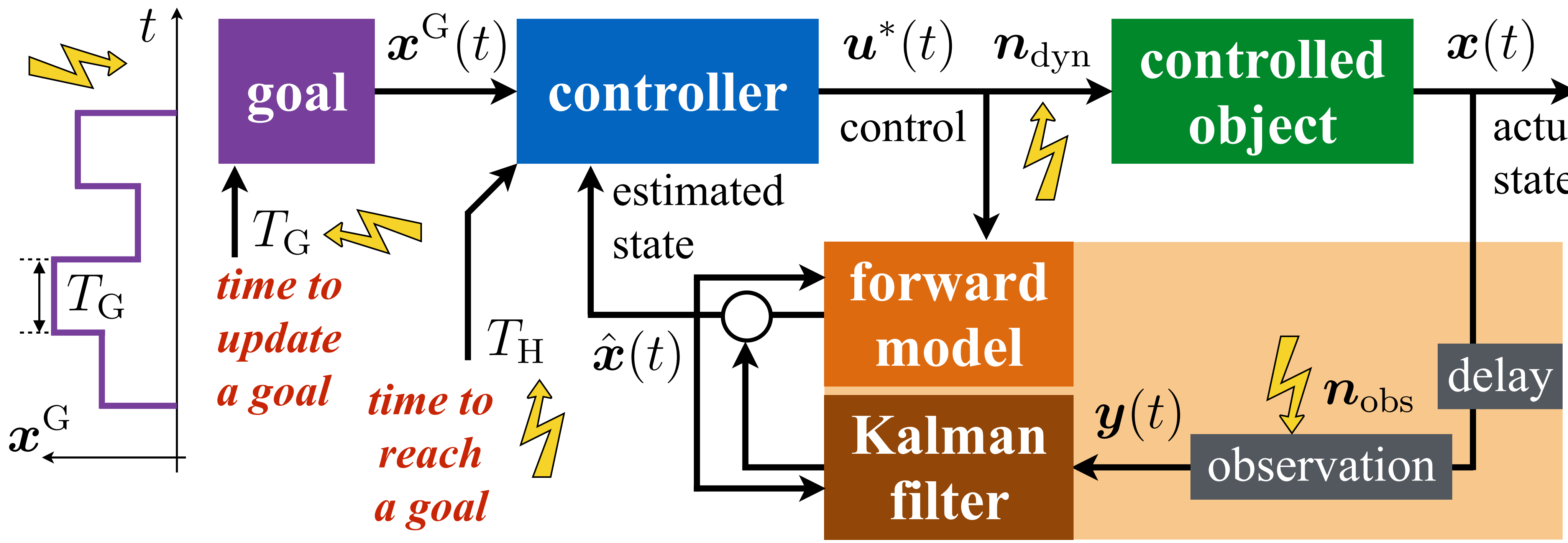
- **Task representation**

— **series of goals** (via-points) played at a fixed and unique frequency (***Goal update time/frequency***)

— Guigon et al., 2019, *J Neurophysiol* 121:715

— Guigon, 2023, *Psychol Rev* 130:23

FORMAL MODEL



$T_{\mathrm{G}} = 0.13$ s

$F_{\mathrm{G}} \approx 8$ Hz

$T_{\mathrm{H}} = 0.28$ s

Control

$$\forall t \ \underset{[t;\,t+T_{\mathrm{H}}]}{\boldsymbol{U}} = \arg\min_{\boldsymbol{u}} \int_t^{t+T_{\mathrm{H}}} L(\boldsymbol{x}(\xi), \boldsymbol{u}(\xi))\, d\xi$$

$$\boldsymbol{u}^*(t) = \boldsymbol{U}(t)$$

$$\begin{cases} \boldsymbol{x}(t) = \hat{\boldsymbol{x}}(t) \\ \boldsymbol{x}(t + T_{\mathrm{H}}) = \boldsymbol{x}^{\mathrm{G}}(t) \end{cases}$$ *boundary conditions*

$$\dot{\boldsymbol{x}}(t) = \boxed{\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))} + \boldsymbol{n}_{\mathrm{dyn}}$$

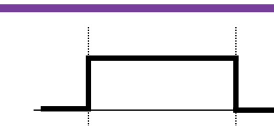
Estimation

$$\dot{\hat{\boldsymbol{x}}}(t) = \boxed{\boldsymbol{f}(\hat{\boldsymbol{x}}(t), \boldsymbol{u}(t))} + \boxed{\mathbf{K}\,(\boldsymbol{y}(t) - \mathbf{H}\hat{\boldsymbol{x}}(t))}$$

Observation

$$\boldsymbol{y}(t) = \mathbf{H}\boldsymbol{x}(t) + \boldsymbol{n}_{\mathrm{obs}}$$

Goal



$$\boldsymbol{x}^{\mathrm{G}}(t) = \sum_{k=1}^{N} \boldsymbol{x}_k^{\mathrm{G}} \ \mathrm{boxcar}(t, (k-1)T_{\mathrm{G}}, kT_{\mathrm{G}})$$

CONTROLLED OBJECT

2D inertial point actuated by linear muscles

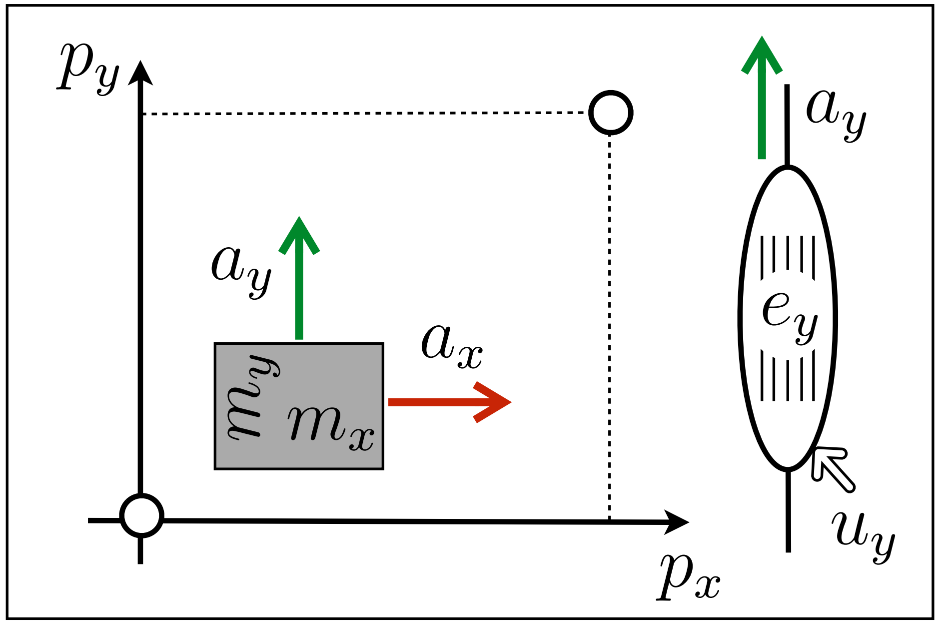
— 1st order linear dynamics

$$
\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}(t)) = \begin{cases} \dot{p}_x = v_x \\ \dot{p}_y = v_y \\ m_x \dot{v}_x = a_x \\ m_y \dot{v}_y = a_y \\ \tau \dot{a}_x = -a_x + e_x \\ \tau \dot{a}_y = -a_y + e_y \\ \tau \dot{e}_x = -e_x + u_x \\ \tau \dot{e}_y = -e_y + u_y \end{cases}
$$

state

position	p
velocity	v
activation	a
excitation	e

$$
\frac{d\boldsymbol{x}}{dt} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u}
$$



Cost function

$$
L(\boldsymbol{x}, \boldsymbol{u}) = u_x^2 + u_y^2
$$

— Kirk, 2004, *Optimal Control Theory*, Dover

— analytical solution using the calculus of variations

— Guigon et al., 2008, *J Comput Neurosci* 24:57

INHERITED RESULTS

The model inherits properties of previous optimal control models

— trajectory formation

— coordination (solution to the degrees-of-freedom problem)

— structure of variability (uncontrolled manifold)

— response to perturbations: flexibility in space and time

— Todorov & Jordan, 2002, *Nat Neurosci* 5:1226

— Guigon et al., 2008, *Eur J Neurosci* 27:1003

— Guigon et al., 2007, *J Neurophysiol* 97:331

— Rigoux & Guigon, 2012, *PLoS Comput Biol* 8:e1002716

NEW RESULTS

Fastest movements
— smoothness and isochrony

Slow movements
— segmentation

Drawing movements
— isochrony, power laws

Discrete vs rhythmic movements
— dwell time, harmonicity

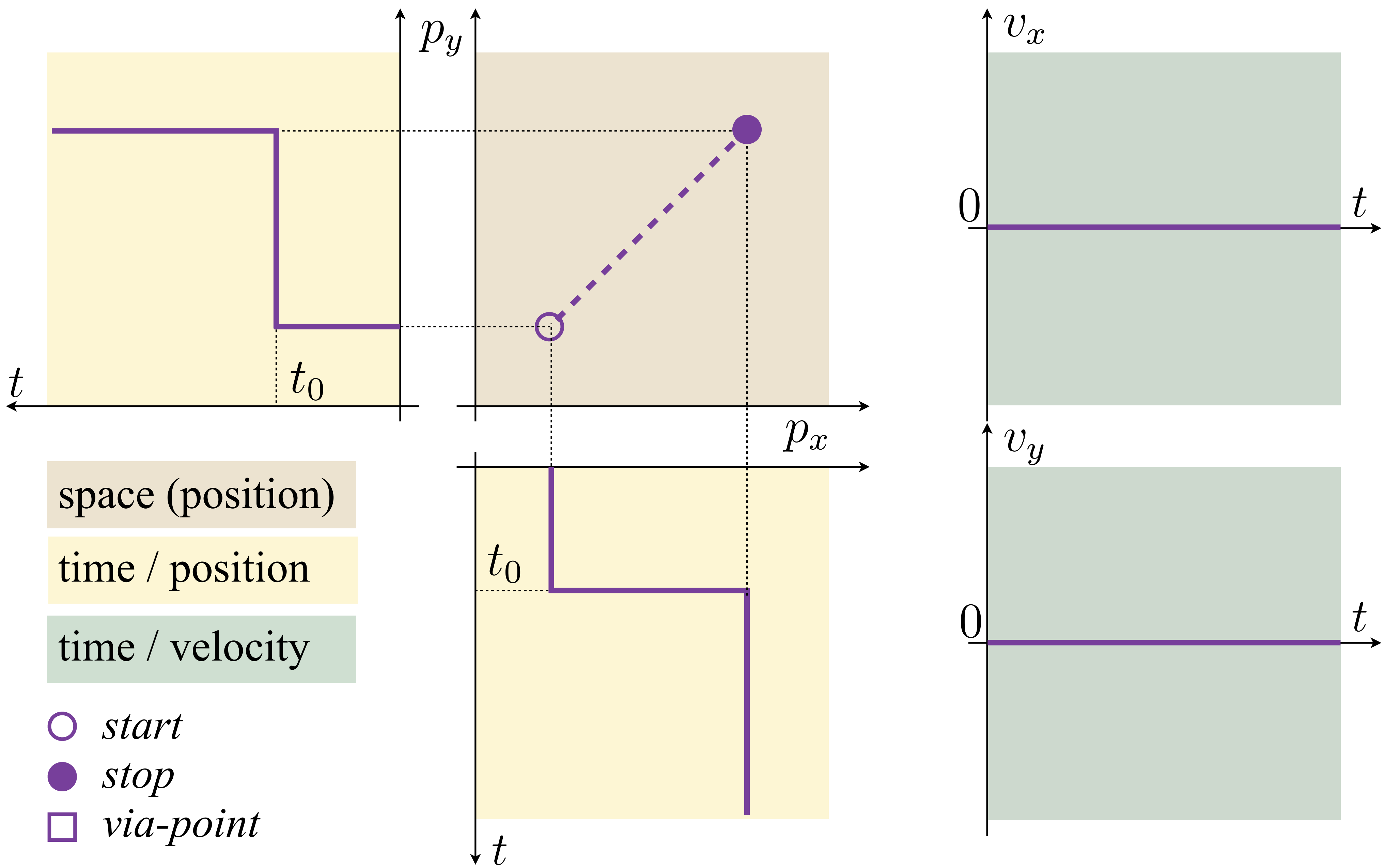
Fitts' law
— rhythmic or discrete

— Guigon et al., 2019, *J Neurophysiol* 121:715

— Guigon, 2023, *Psychol Rev* 130:23

FAST(EST) MOVEMENTS

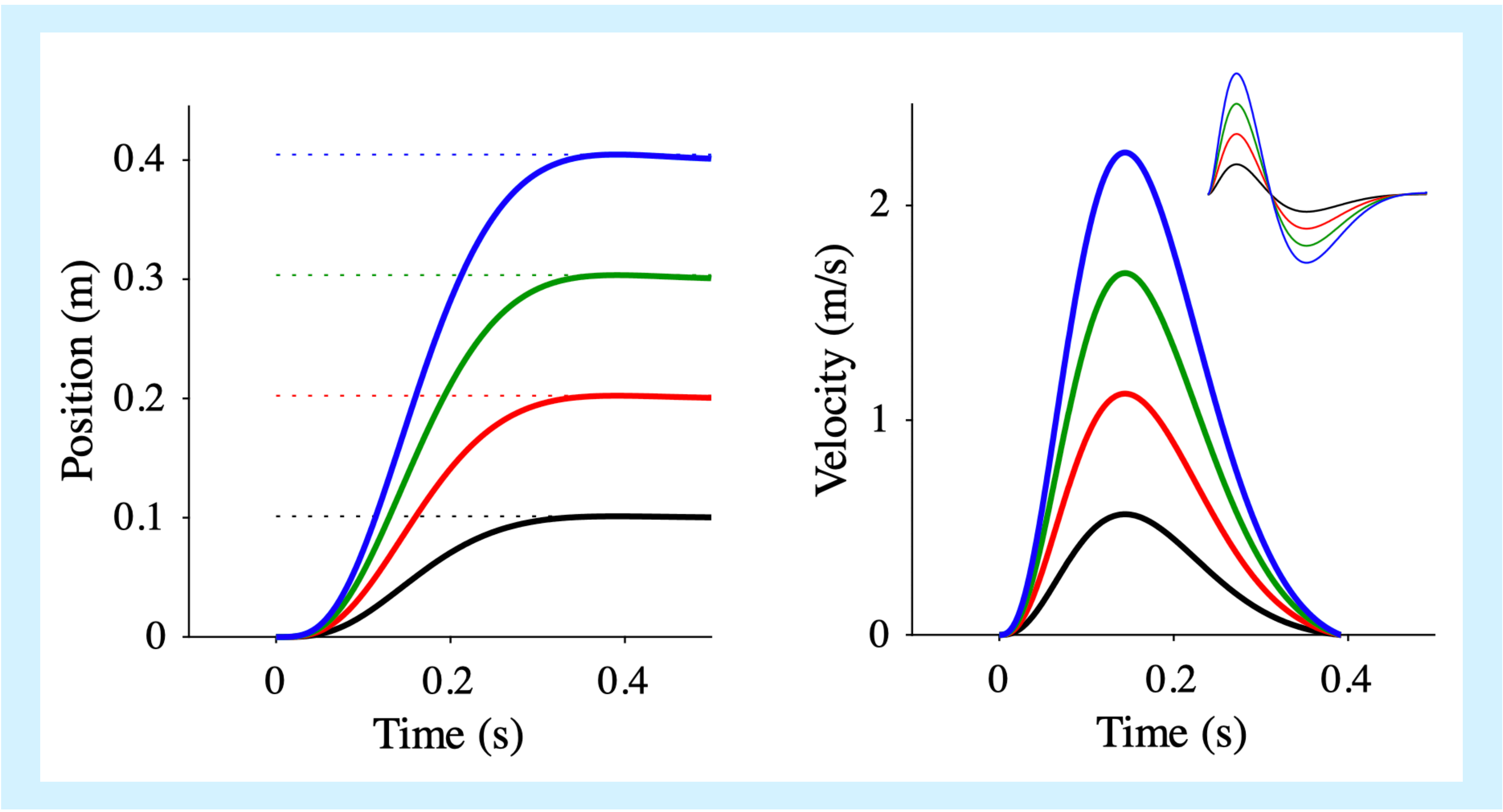
Task representation



FAST(EST) MOVEMENTS

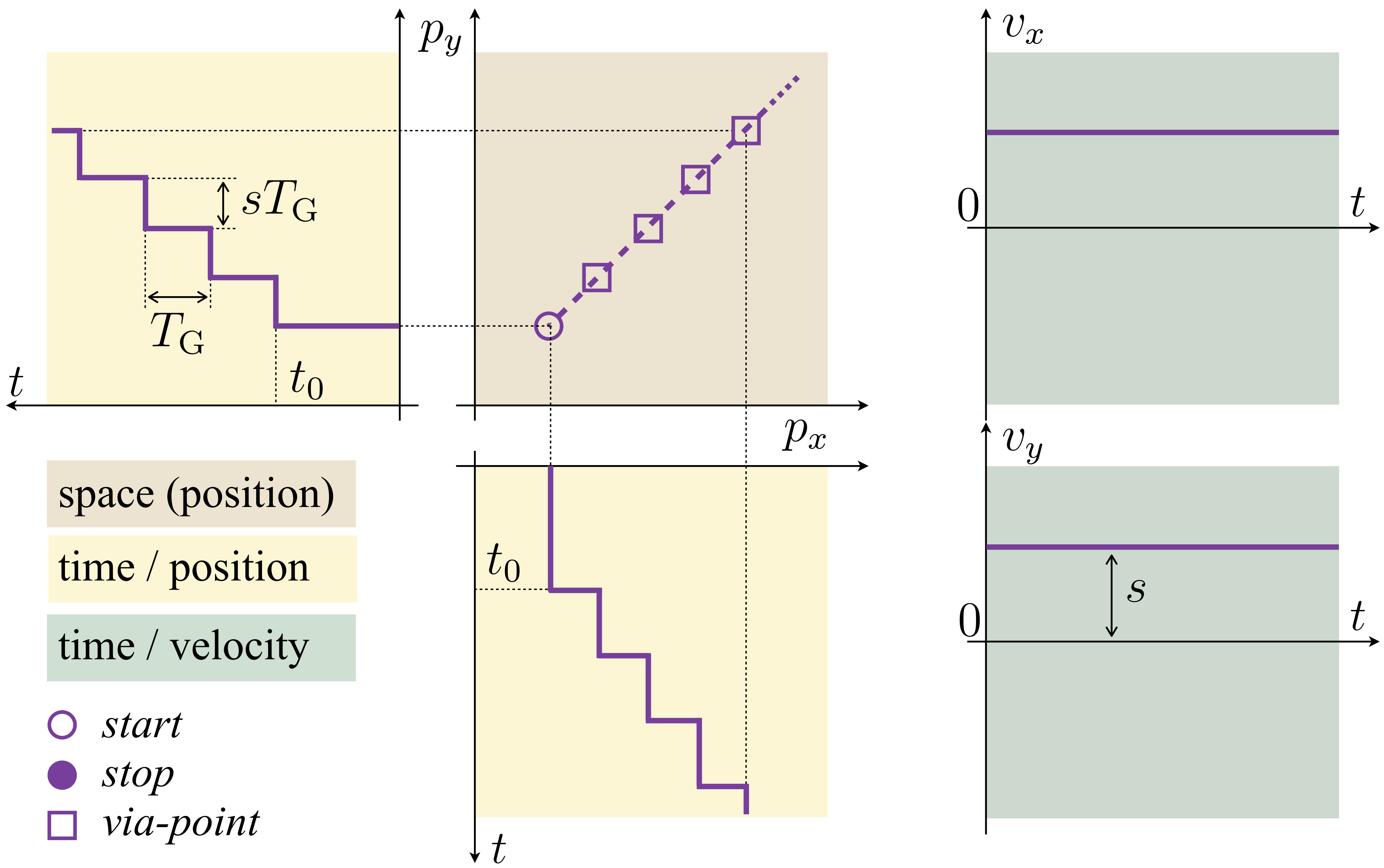
Smoothness and isochrony

— there exists a unique class of smooth movements whose duration is defined by the receding horizon

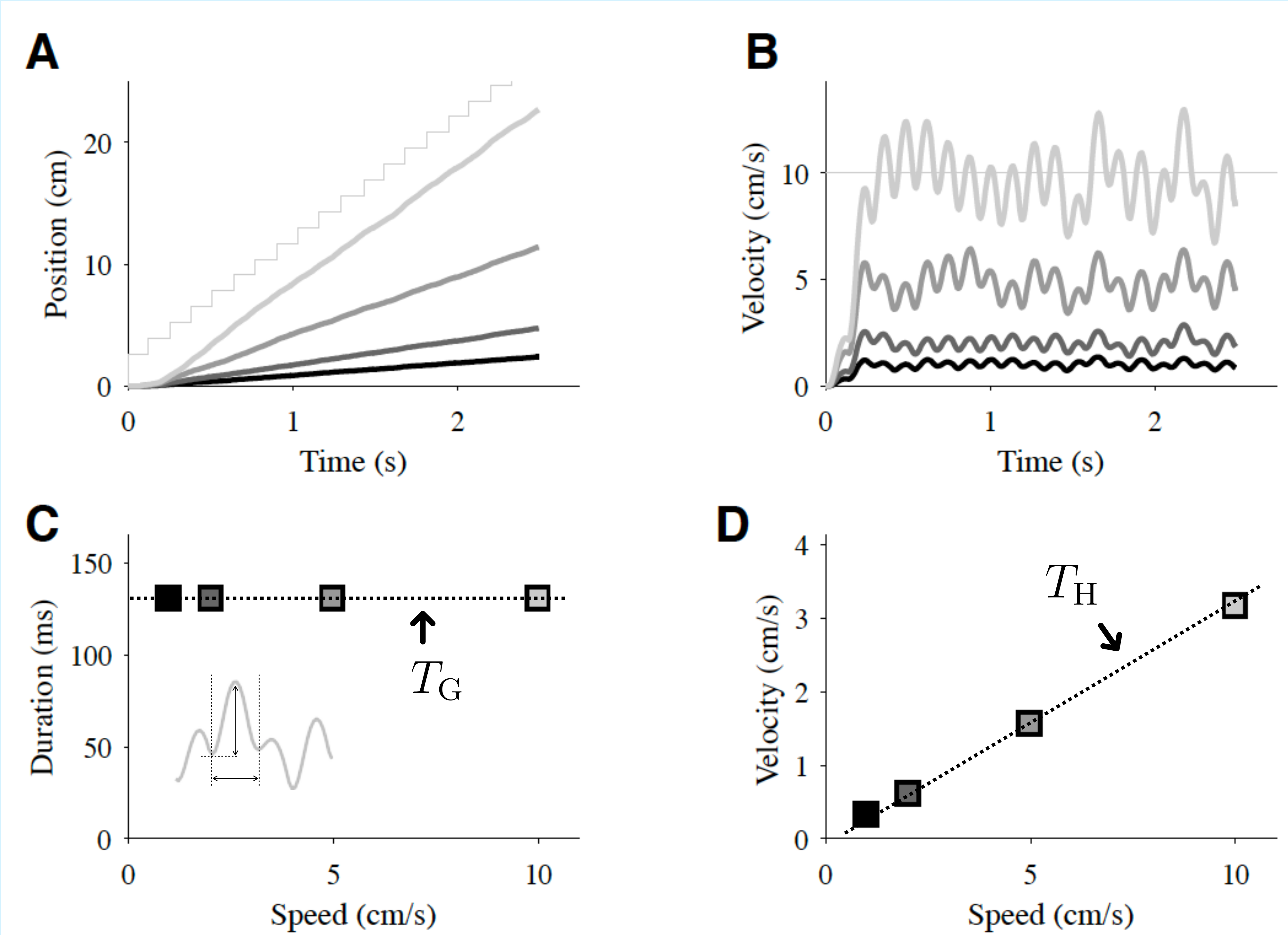


SLOW MOVEMENTS

Task representation



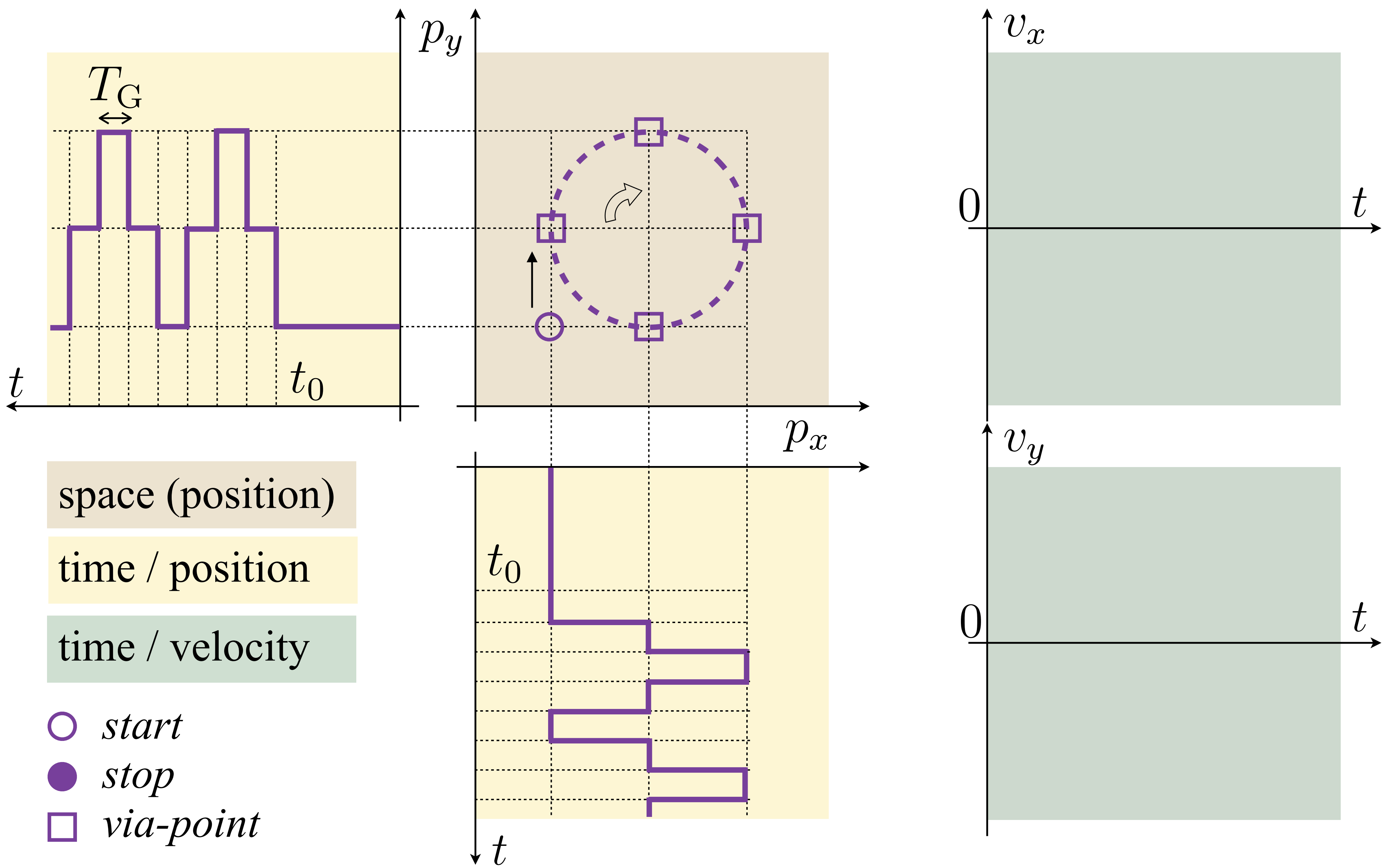
SLOW MOVEMENTS



— Guigon et al., 2019, *J Neurophysiol* 121:715

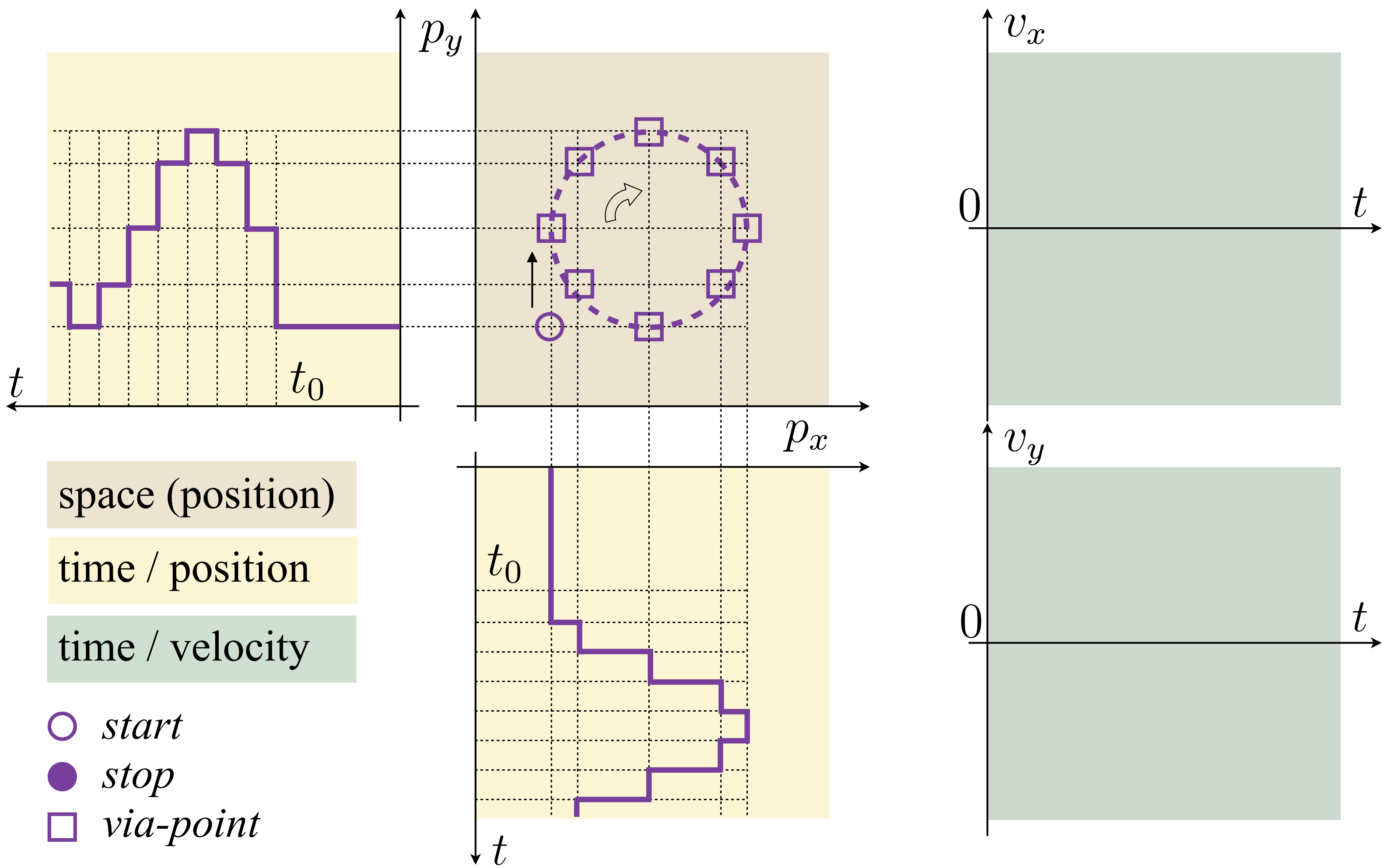
DRAWING MOVEMENTS

Task representation

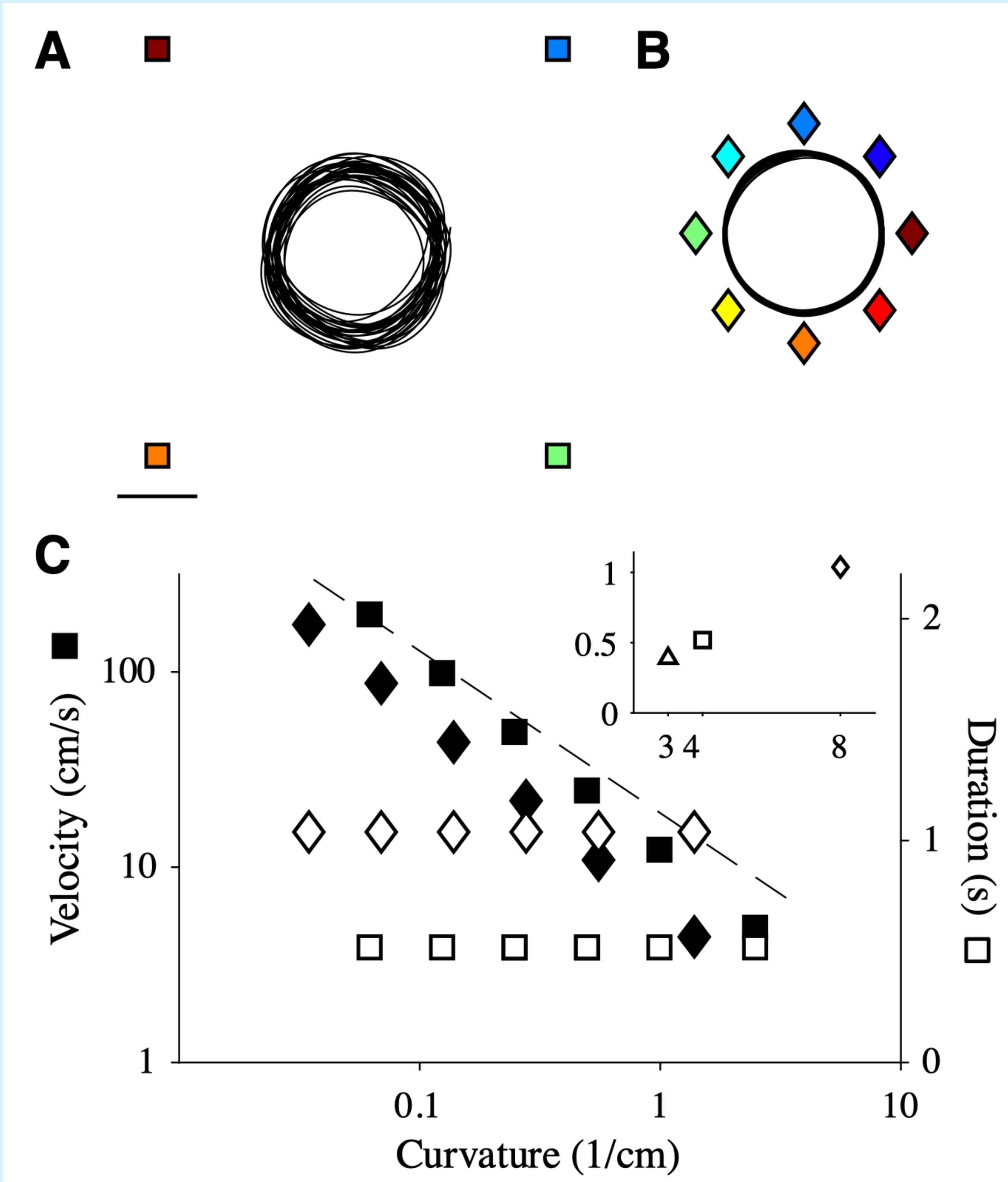


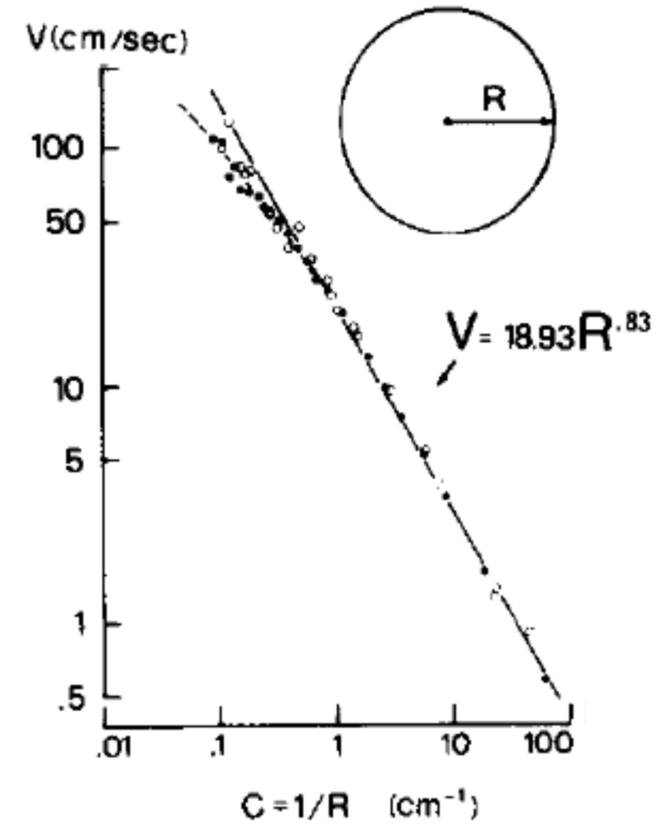
DRAWING MOVEMENTS

Task representation



DRAWING MOVEMENTS — CIRCLE





—Viviani & McCollum, 1983, *Neuroscience* 10:211

DRAWING MOVEMENTS — POWER LAWS

Power laws

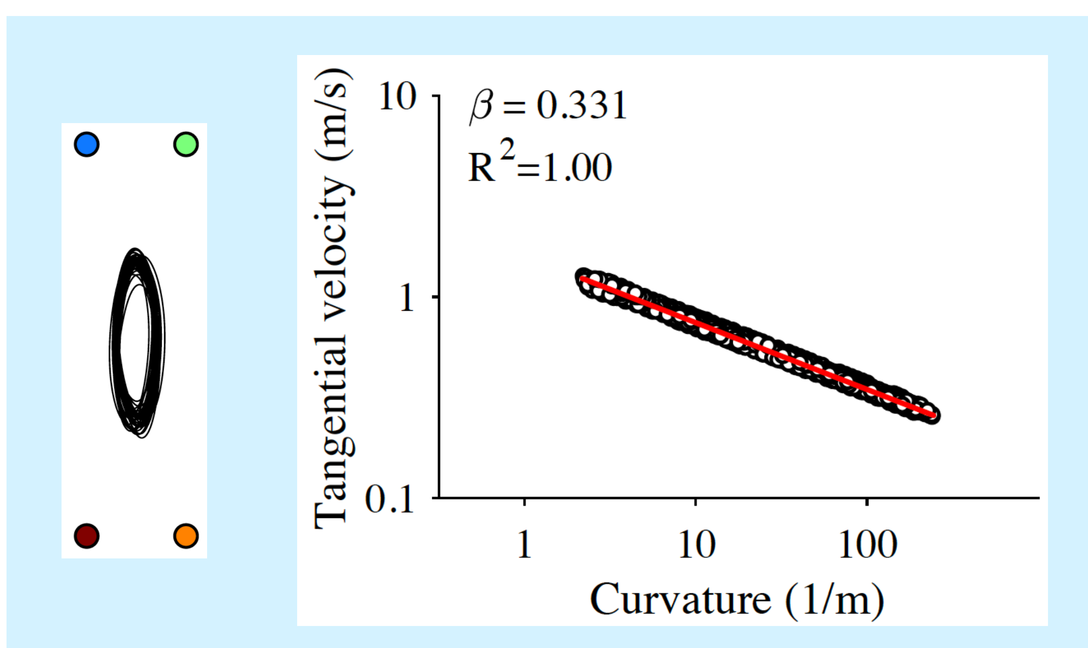
1/3 and 2/3 power laws (scribbling, ellipses)

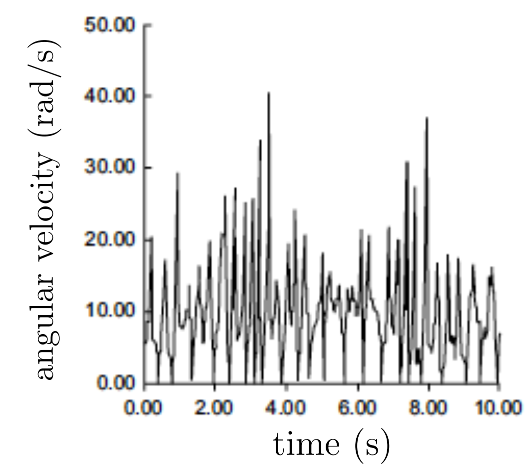
$$v(t) \propto \kappa(t)^{-\beta} \qquad \beta \approx \frac{1}{3}$$

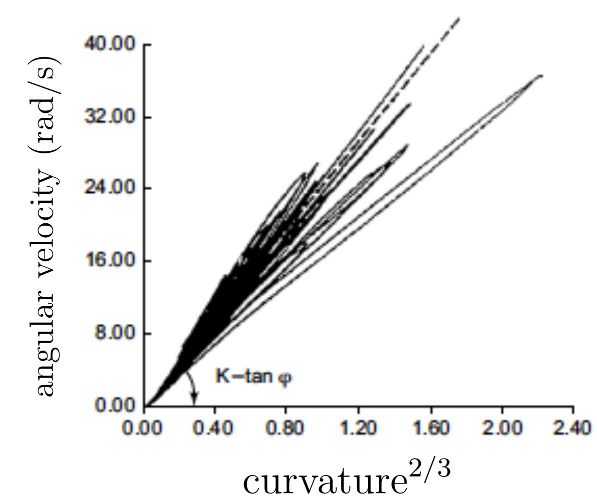
$$v(t) \propto r(t)^{\beta}$$

$$a(t) \propto \kappa(t)^{1-\beta}$$

$v(t)$	tangential velocity
$a(t)$	angular velocity
$\kappa(t)$	curvature
$r(t)$	radius of curvature

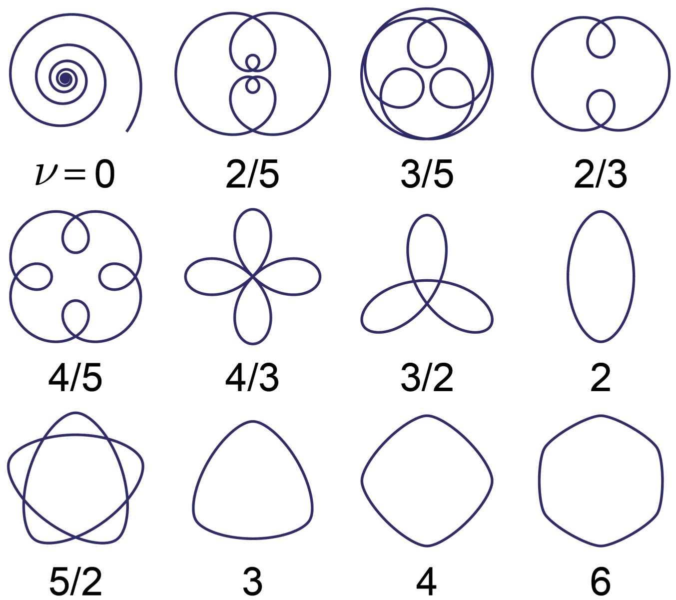


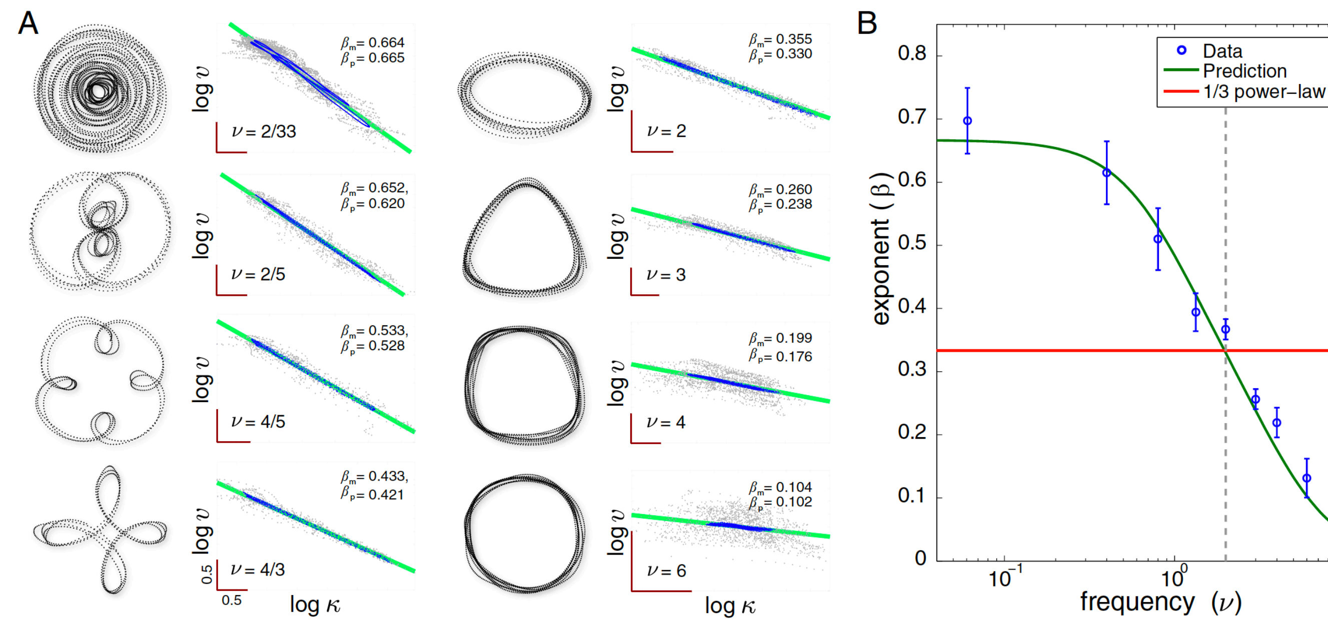




— Lacquaniti et al., 1983, *Acta Psychol* 54:115

MULTIPLE POWER LAWS

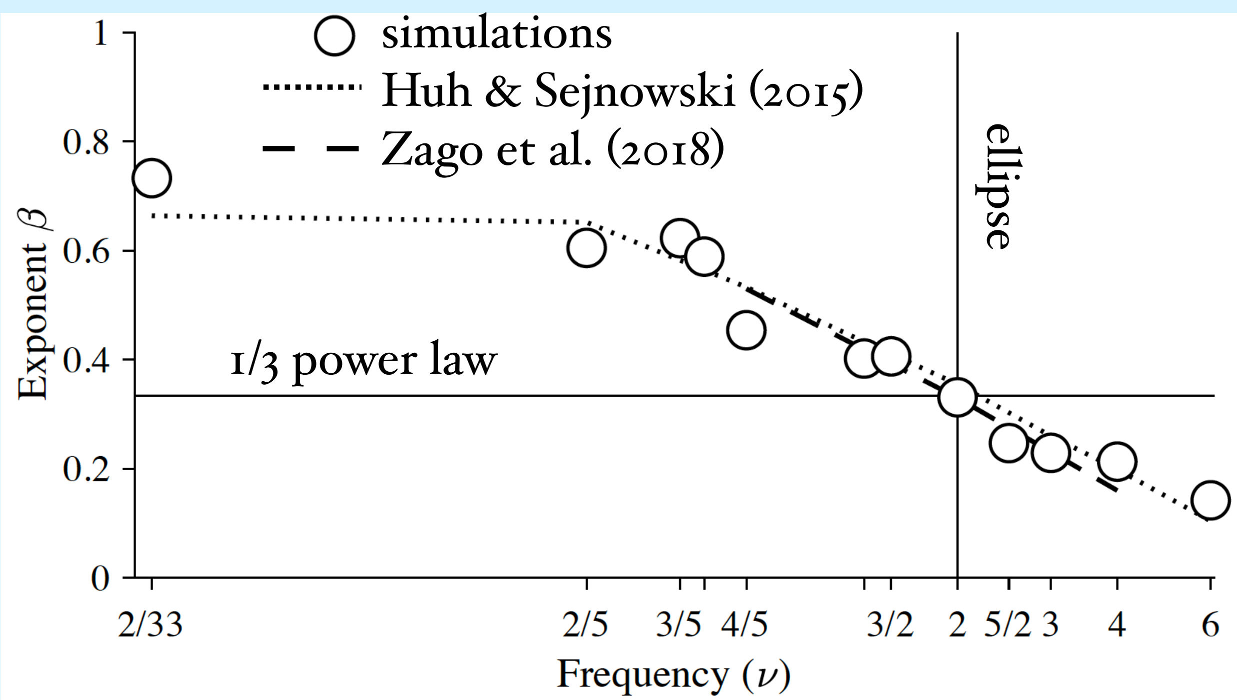




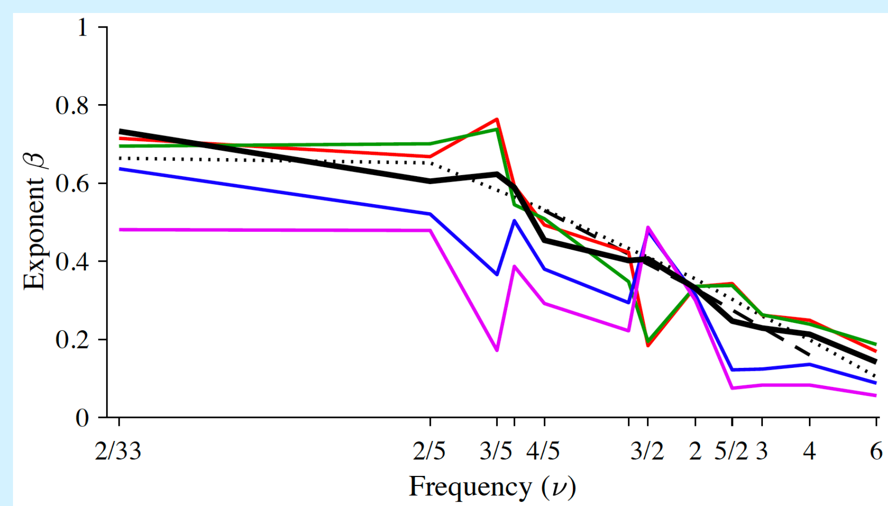
$$\log r(\theta) = \epsilon \sin(\nu\theta) \qquad v(t) \propto \kappa(t)^{-\beta}$$

— Huh & Sejnowski, 2015, *PNAS* 112:E3950

— Zago et al., 2018, *Exp Brain Res* 236:69



T_H	0.28	0.38	0.48	0.28	0.28
T_G	0.13	0.13	0.13	0.18	0.23



NEURONAL IMPLEMENTATION

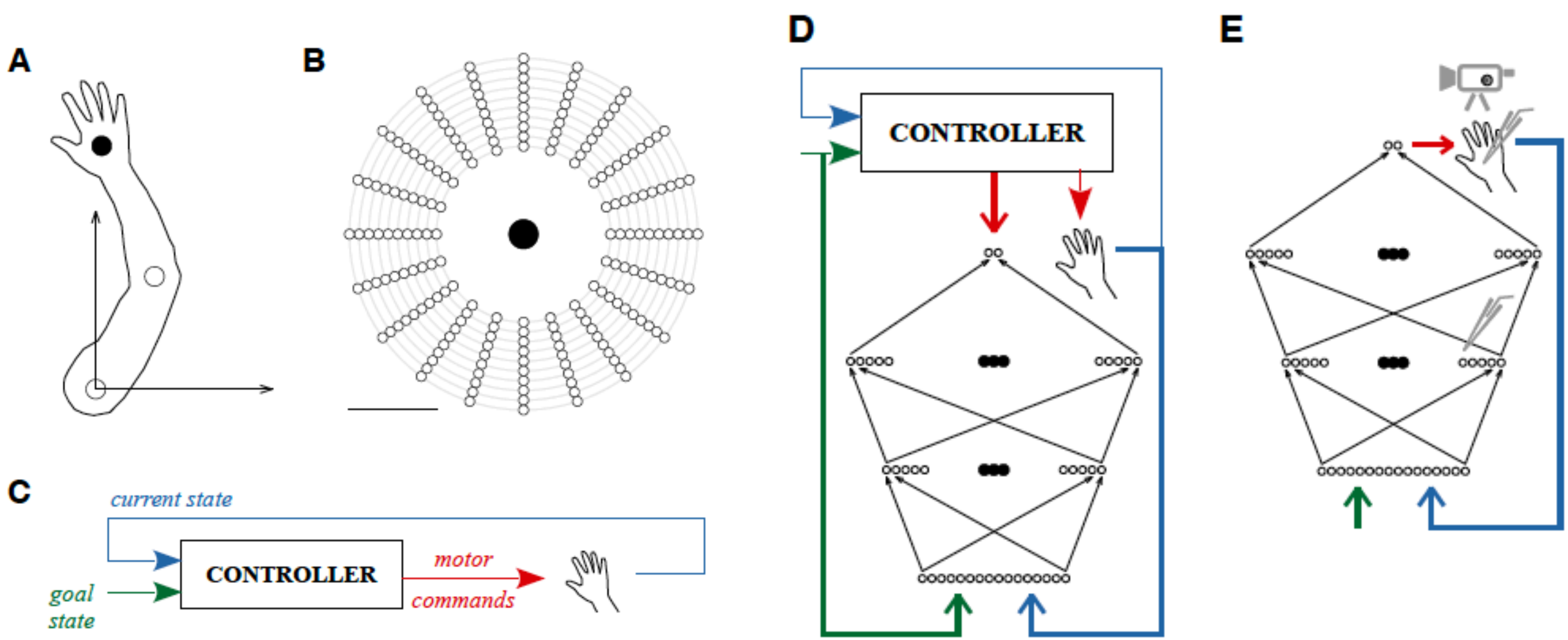
- **Linear case**

— linear control policy

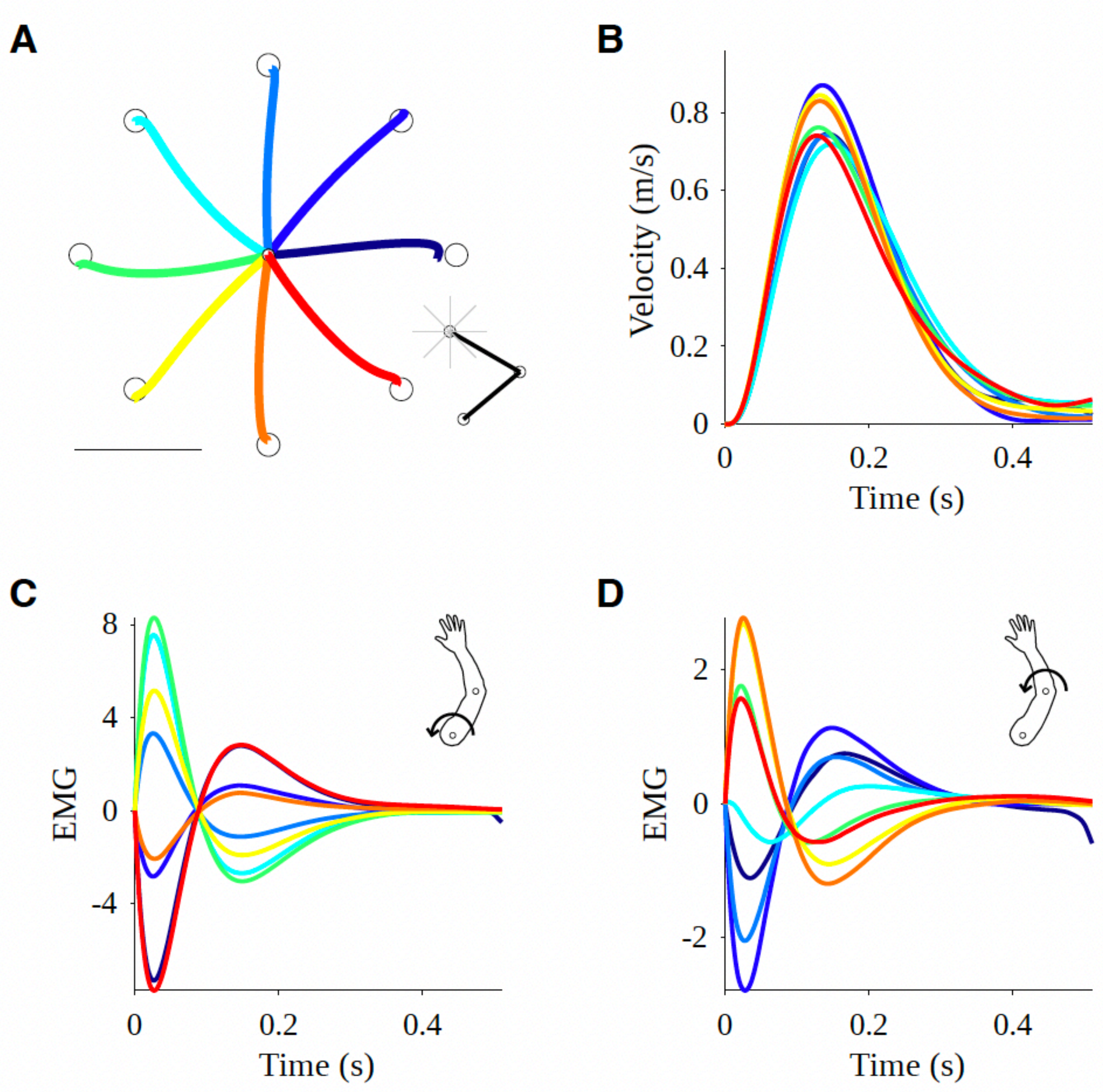
$$\text{motor commands} = \alpha \ \text{current state} + \beta \ \text{goal state}$$

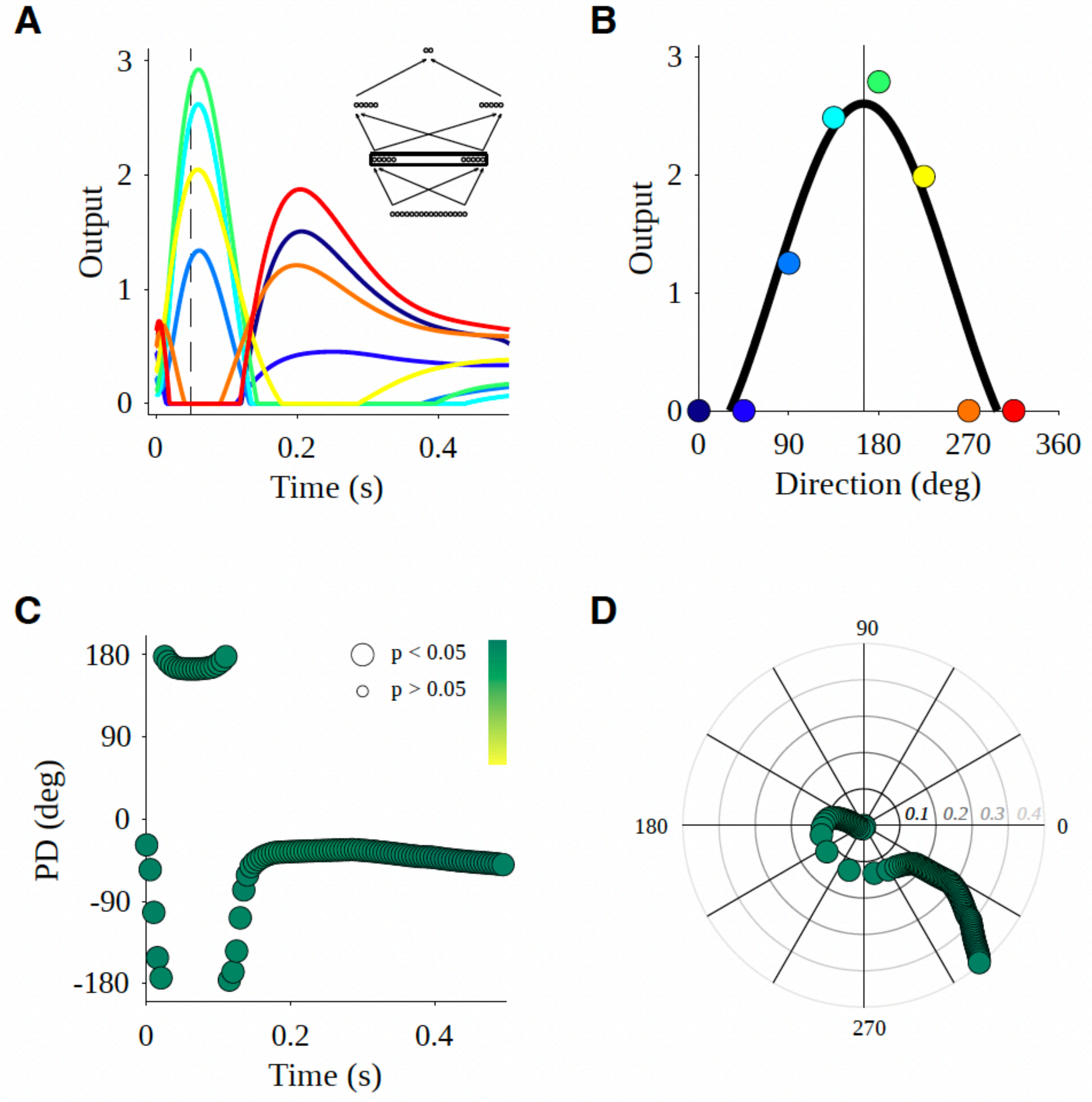
- **Nonlinear case**

— nonlinear neural network trained to approximate a controlller

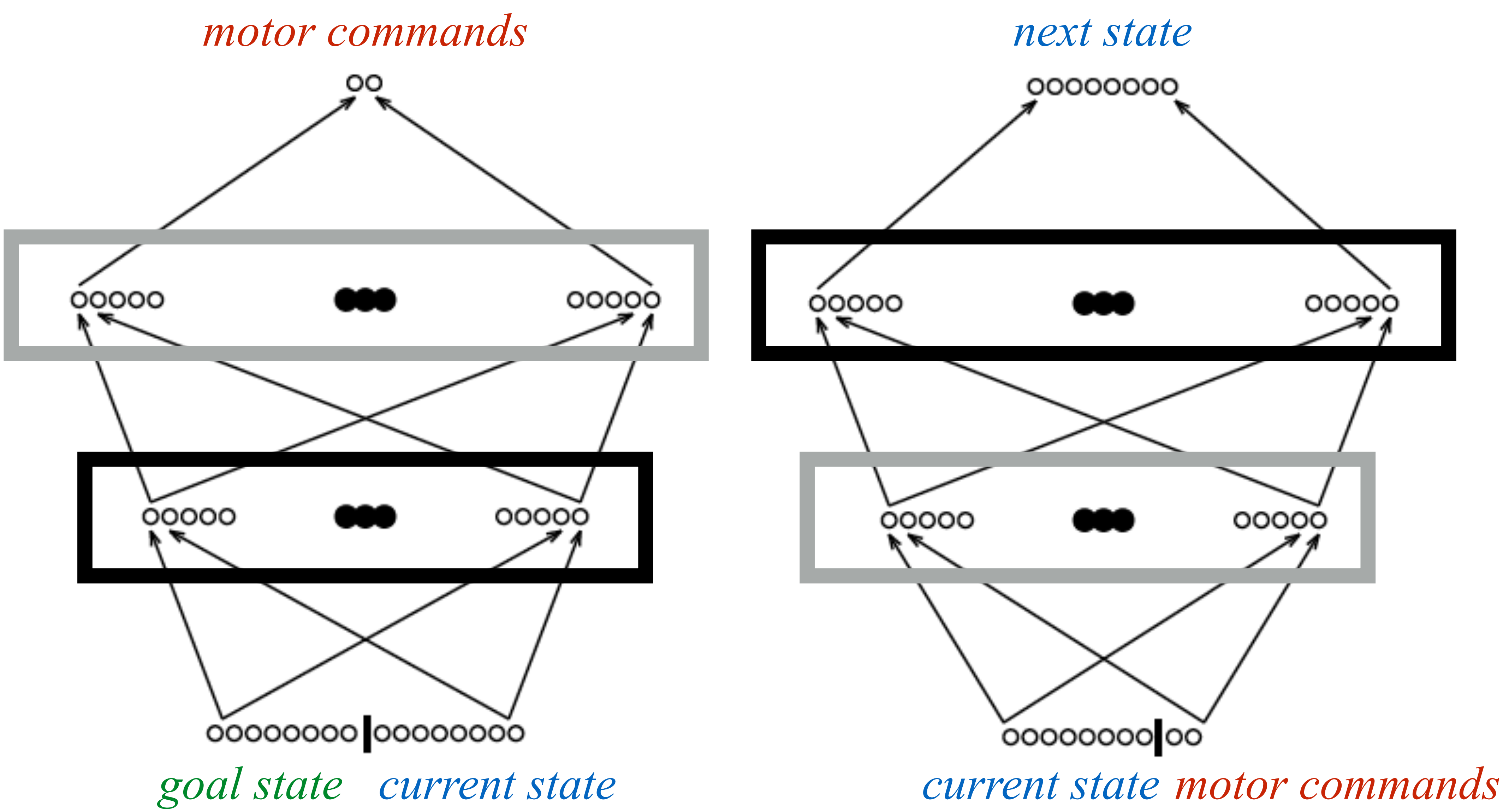


NEURONAL IMPLEMENTATION





CONTROLLER VS ESTIMATOR

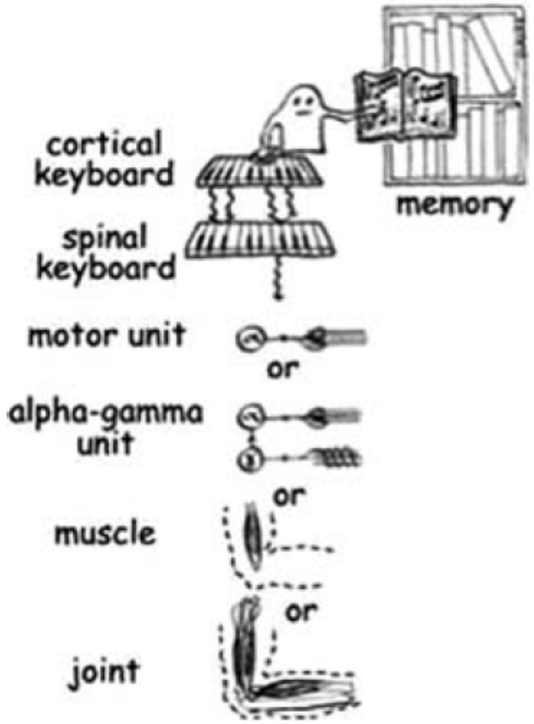


no inactive neurons 98% neurons: tuned tuning: 87.2% of time	40% inactive neurons 36% neurons: tuned tuning: 5.4% of time	68% inactive neurons 31% neurons: tuned tuning: 85% of time	54% inactive neurons 41% neurons: tuned tuning: 70% of time

DISCUSSION

Control theory approach

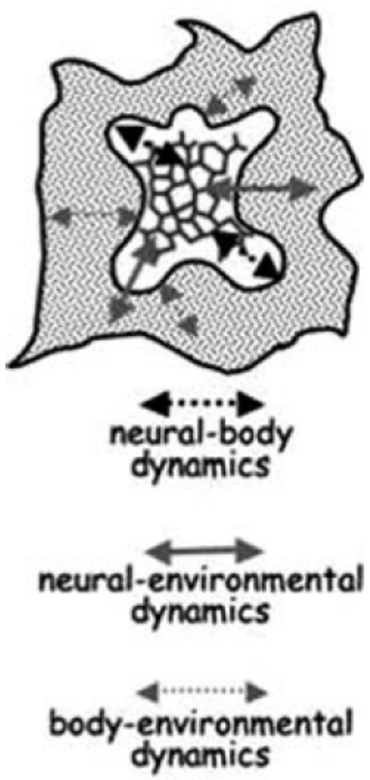
— the nervous system performs computations
— two parts in the body: a controller (nervous system?) and a controlled object
— actions are represented and stored in the nervous system



Information processing — Cognitive — Motor programs

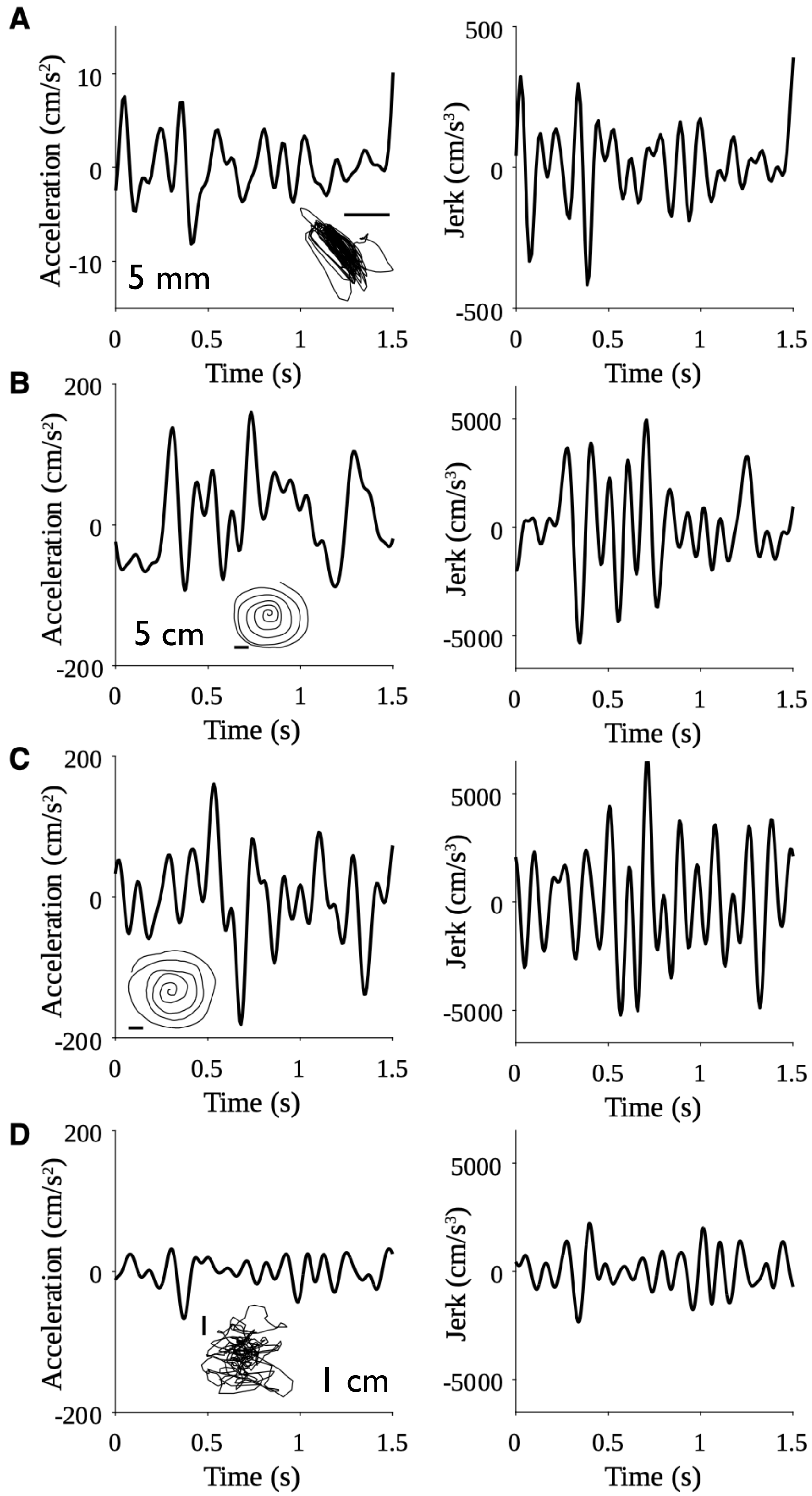
Physical approach

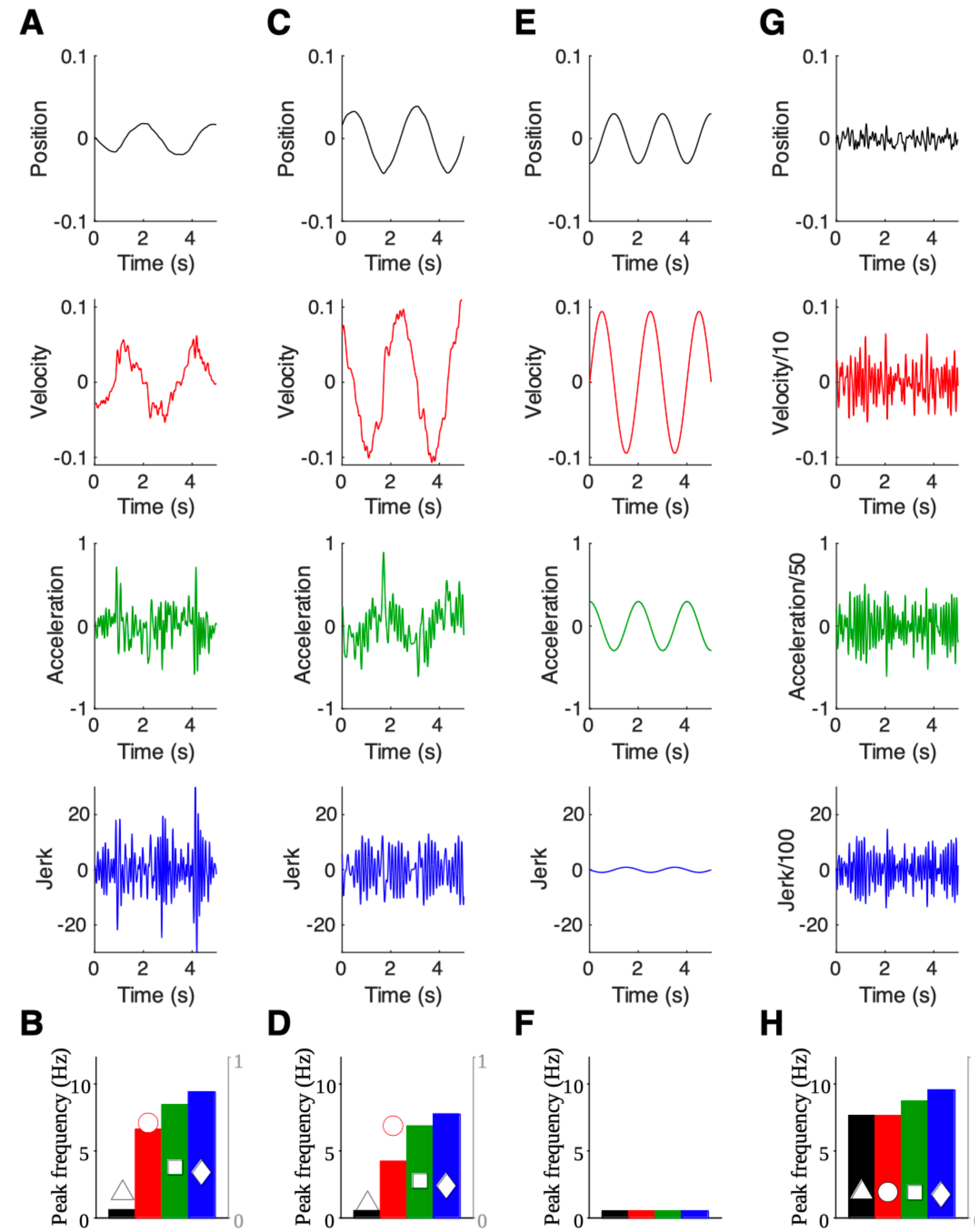
— the nervous system does not perform computations
— actions are not *represented* in the brain by way of a plan or a program but emerge naturally (or self-organize) out of the physical properties of the body, the environment and the brain



Action systems — Dynamical systems — Task dynamics

COMMON CONTENT OF ACTIONS





— Apthorp et al., 2014, *PLoS One* 9:e113897

— Drotár et al., 2016, *Artif Intell Med* 67:39

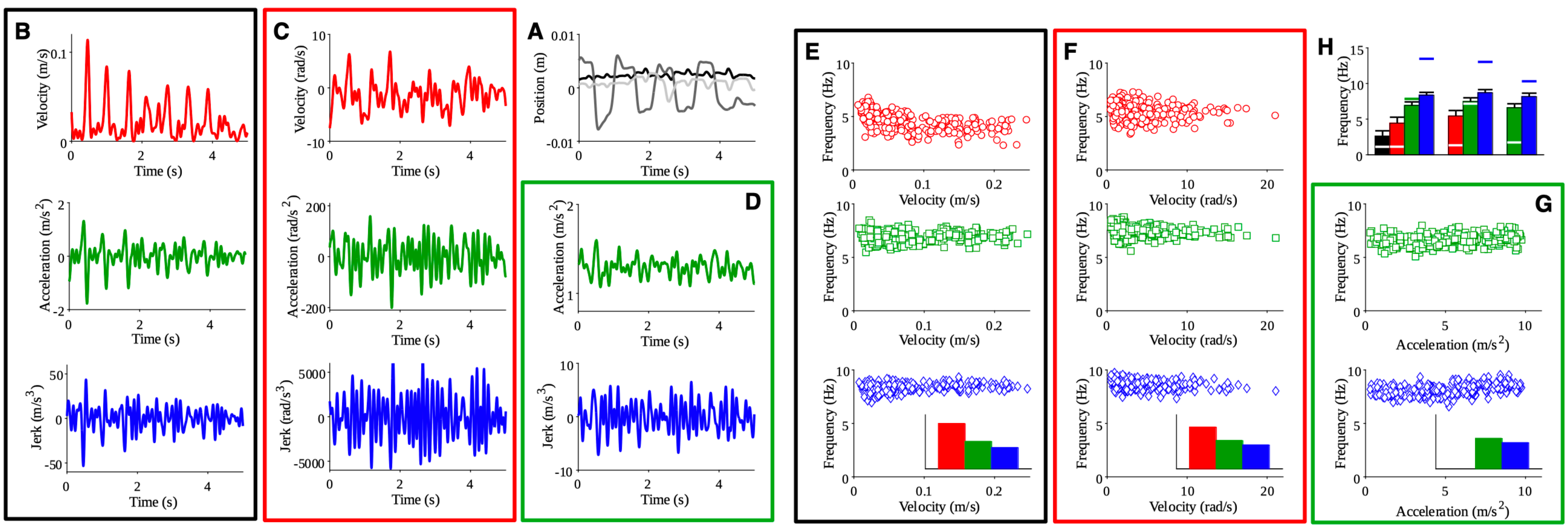
— Kuberski & Gafos, 2019, *PLoS One* 14:e0213851

— Guigon et al., 2019, *J Neurophysiol* 121:715

COMMON CONTENT OF ACTIONS

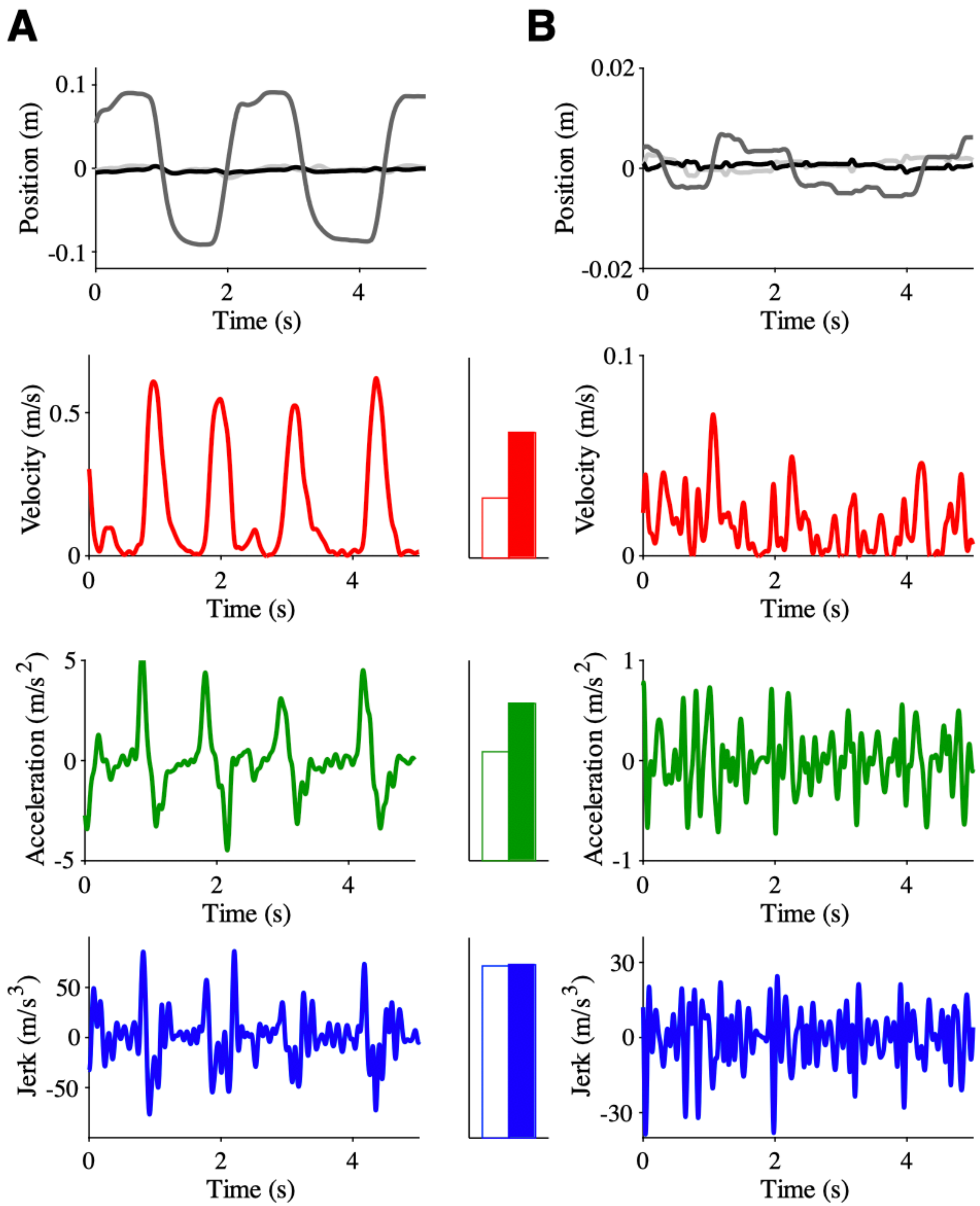
Continuous actions

reciprocal Fitts' like task



—Wang & Majewicz Fey, 2018, *PLoS One* 13:e0195053

COMMON CONTENT OF ACTIONS

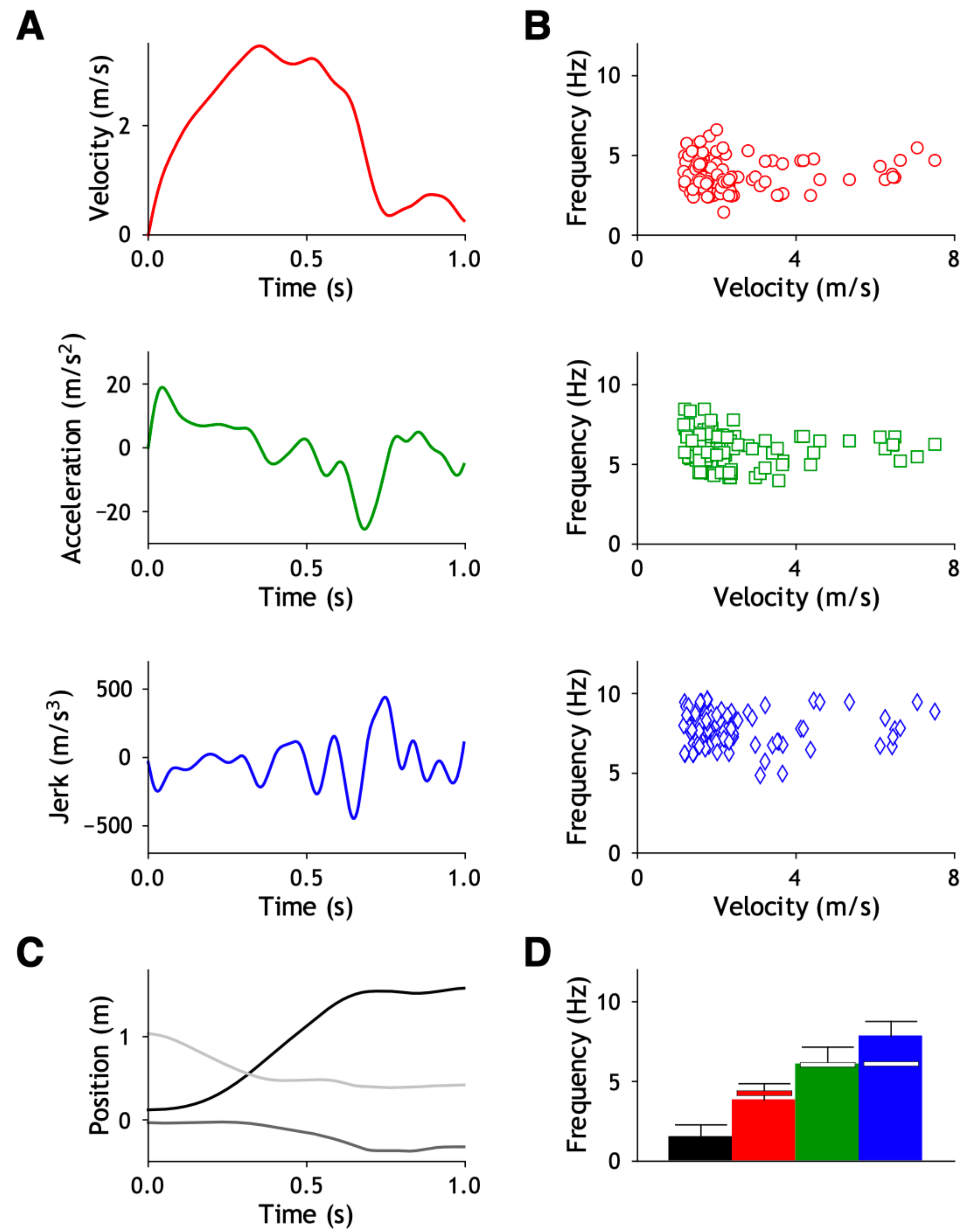


—Wang & Majewicz Fey, 2018,
PLoS One 13:e0195053

COMMON CONTENT OF ACTIONS

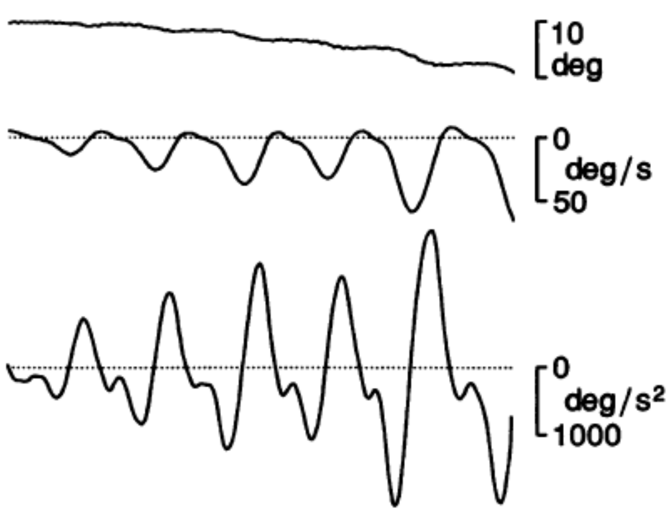
Discrete actions

throwing movements

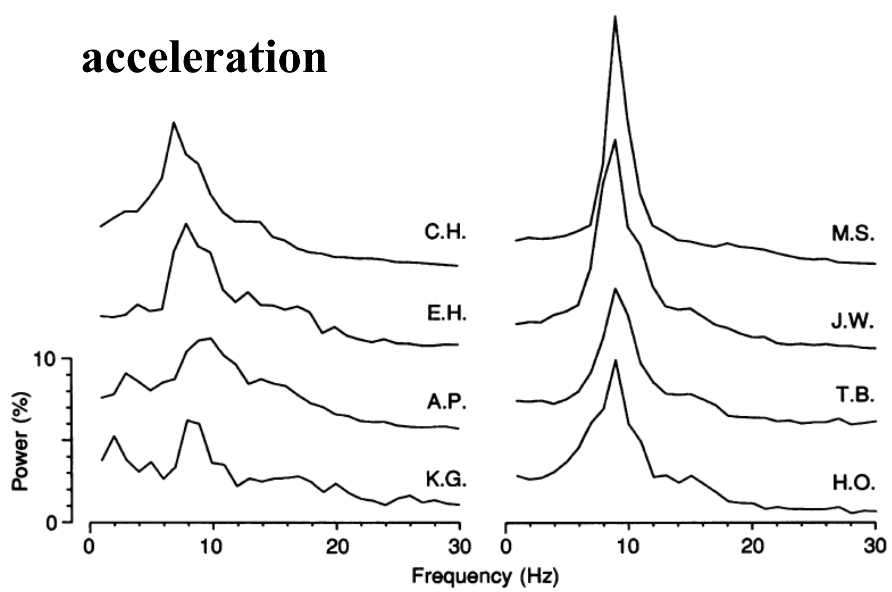


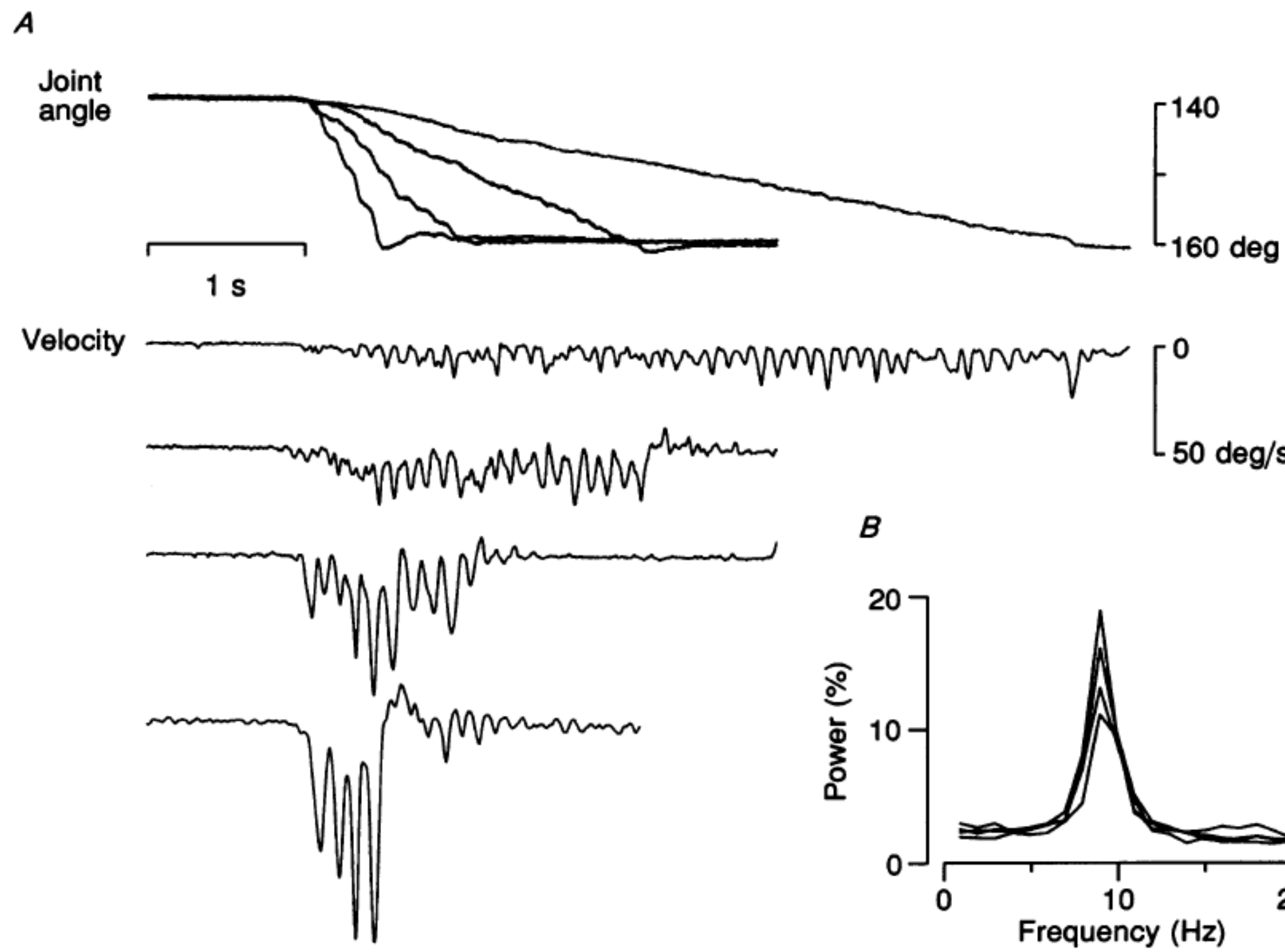
— O'Connell et al., 2021, *SportRxiv*

COMMON CONTENT OF ACTIONS



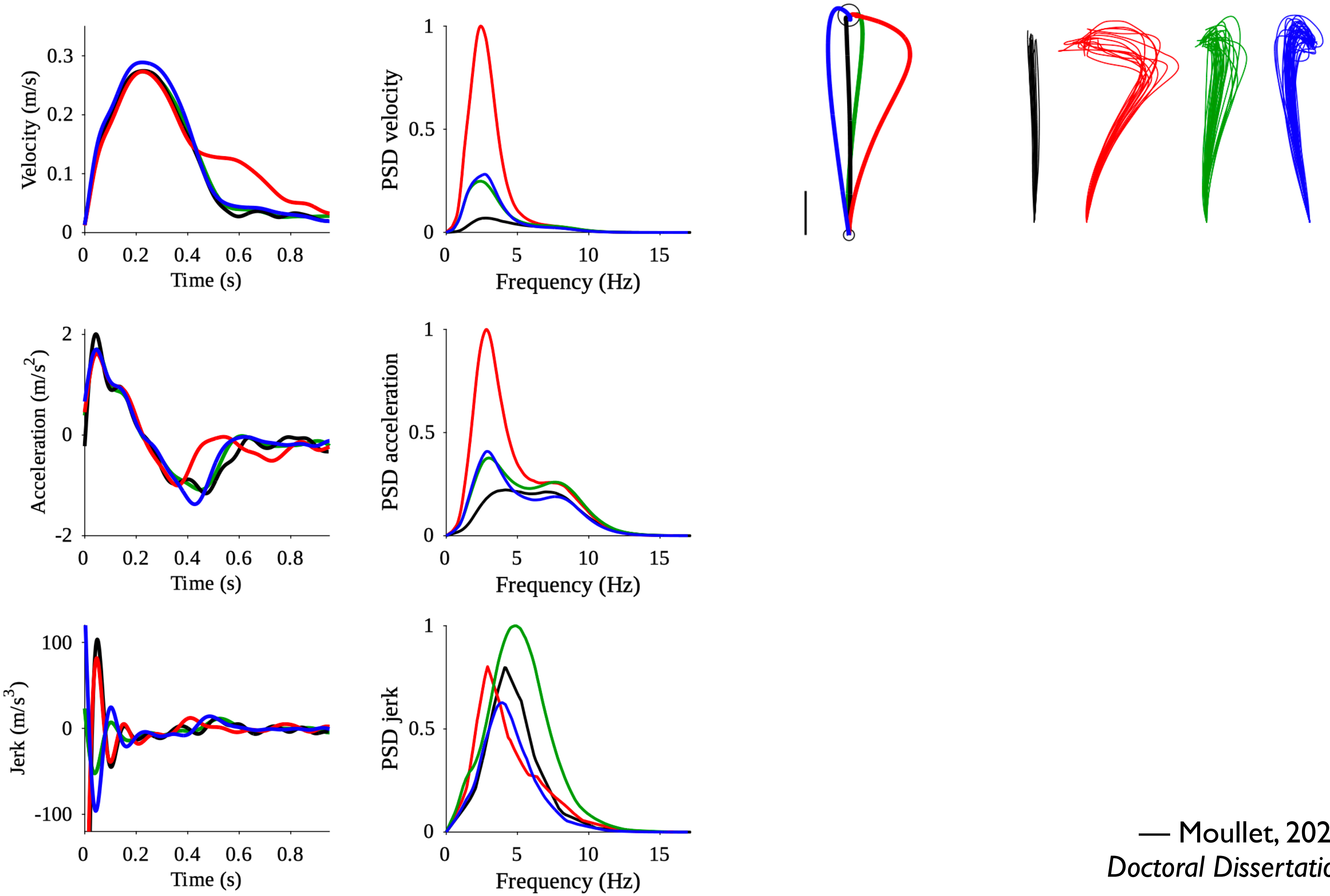
acceleration





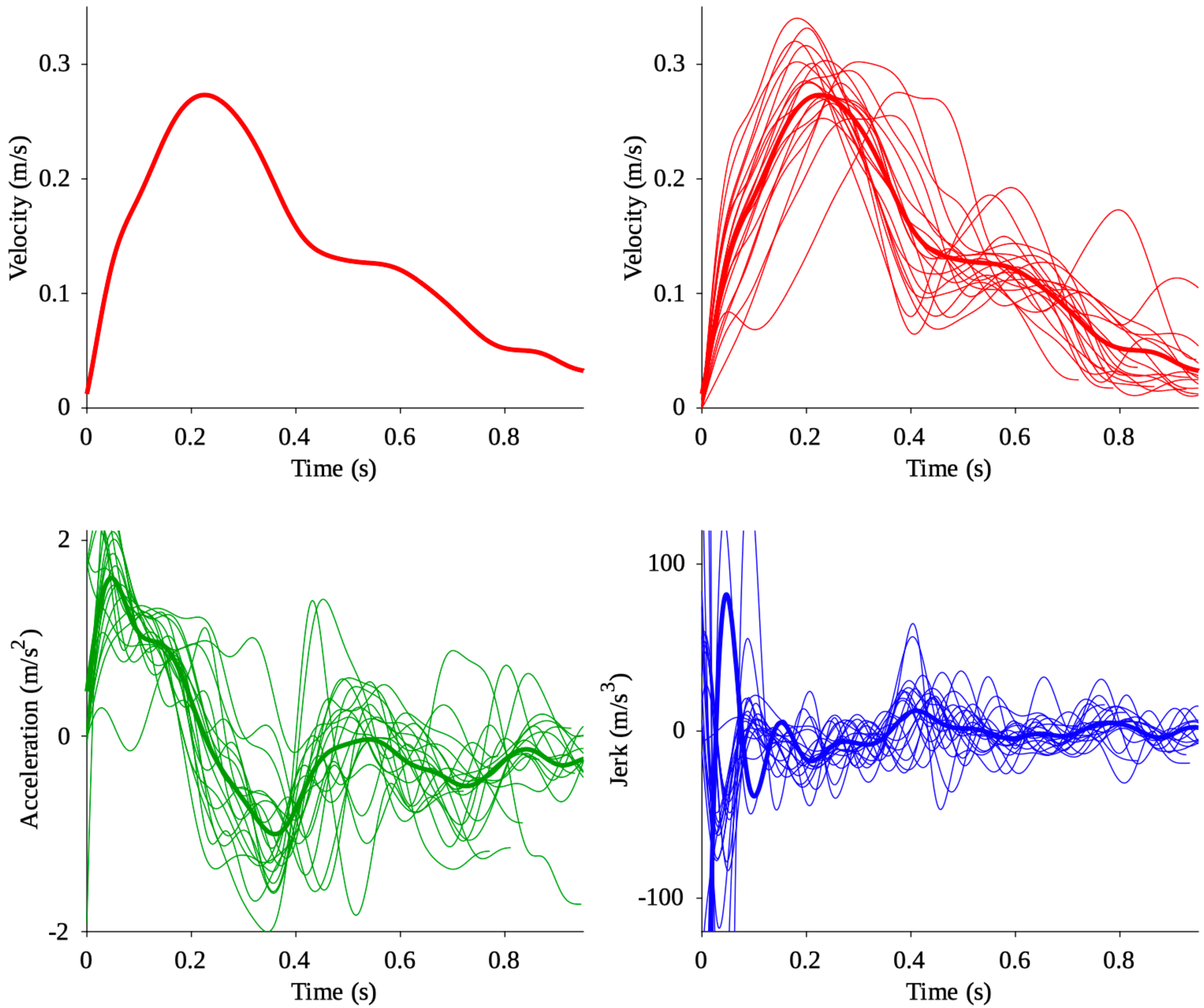
—Vallbo & Wessberg, 1993,
J Physiol (Lond) 469:673

COMMON CONTENT OF ACTIONS



— Moullet, 2022,
Doctoral Dissertation

COMMON CONTENT OF ACTIONS



— Moullet, 2022, *Doctoral Dissertation*