Geometry and topology of manifolds **2010-2011**

Objectives

To relate properties of topological manifolds with properties of differentiable manifolds. To compare singular cohomology with De Rham cohomology of differentiable manifolds. To know Poincaré duality and some of its consequences. To know the principles of Morse theory and some of its applications. To calculate characteristic classes of vector bundles over compact manifolds and describe their use. To prove the existence of exotic spheres.

Assessment of learning outcomes

The completion of exercises during the course will be assessed for a total of 20% of the final grade and the presentation and defence of the assignment on a topic in the unit or on a closely related topic will be assessed for 80%. The evaluation will consider both the quality of the student's writing in the assignment and the student's precision in the presentation of the assignment and exercises. Those students not wishing to follow the system of continuous assessment will be evaluated in a single examination at the end of the lecture period.

Teaching blocks

1. Singular cohomology
2. Differential forms and De Rham cohomology
3. Orientability and duality on manifolds
4. Characteristic classes of vector bundles
5. Morse theory
6. Milnor's exotic spheres

Reading and study resources

R. Bott, L. W. Tu, *Differential Forms in Algebraic Topology*, Graduate Texts in Math. 82, Springer, New York, 1986 (1st ed. 1982).

G. E. Bredon, *Topology and Geometry*, Graduate Texts in Math. 139, Springer, New York, 1993.

M. J. Greenberg, J. R. Harper, *Algebraic Topology: A First Course,* Math. Lecture Note Series 58, Addison-Wesley, Reading, 1981.

A. Hatcher, *Algebraic Topology*, Cambridge University Press, Cambridge, 2002.

M. W. Hirsch, *Differential Topology*, Graduate Texts in Math. 33, Springer, New York, 1994 (1st ed. 1976).

J. W. Milnor, On manifolds homeomorphic to the 7-sphere, *Ann. of Math.* **64** (1956), 399–405.

J. W. Milnor, J. D. Stasheff, *Characteristic Classes*, Ann. of Math. Studies 76, Princeton University Press, Princeton, 1974.

J. W. Vick, *Homology Theory: An Introduction to Algebraic Topology*, Graduate Texts in Math. 145, Springer, New York, 1994 (1st ed. Academic Press, 1973).