Geometry and topology of manifolds **2011-2012**

Objectives

General: To provide topological and differentiable tools for the study of manifolds.

Specific: To compare singular cohomology of spaces with de Rham cohomology of smooth manifolds; to prove Poincaré duality and learn some of its consequences; to know the principles of Morse theory and some of its applications; to operate with characteristic classes of vector bundles over compact manifolds and see their uses; to prove the existence of distinct differentiable structures on the 7-sphere.

Assessment of learning outcomes

The completion of exercises during the course will be assessed for a total of 20% of the final grade, and the presentation and defence of the assignment on a topic in the unit or on a closely related topic will be assessed for the remaining percentage (50% of the grade will be based on the content of the written work and 30% on the quality of the oral presentation). Those students not wishing to follow the system of continuous assessment will be evaluated in a single examination at the end of the lecture period.

Teaching blocks

1. Singular cohomology
2. Differential forms and de Rham cohomology
3. Orientability and duality on manifolds
4. Morse theory
5. Characteristic classes of vector bundles
6. Milnor's exotic spheres

Reading and study resources

R. Bott, L. W. Tu, *Differential Forms in Algebraic Topology*, Graduate Texts in Mathematics 82, Springer, New York, 1986 (1st ed. 1982).

G. E. Bredon, *Topology and Geometry*, Graduate Texts in Mathematics 139, Springer, New York, 1993.

M. J. Greenberg, J. R. Harper, *Algebraic Topology: A First Course,* Mathematics Lecture Note Series 58, Addison-Wesley, Reading, 1981.

A. Hatcher, *Algebraic Topology*, Cambridge University Press, Cambridge, 2002.

M. W. Hirsch, *Differential Topology*, Graduate Texts in Mathematics 33, Springer, New York, 1994 (1st ed. 1976).

J. W. Milnor, On manifolds homeomorphic to the 7-sphere, *Ann. of Math.* **64** (1956), 399–405.

J. W. Milnor, *Morse Theory*, Princeton University Press, Princeton, 1963.

J. W. Milnor, *Lectures on the h-cobordism theorem*, Princeton University Press, Princeton, 1965.

J. W. Milnor, J. D. Stasheff, *Characteristic Classes*, Annals of Mathematics Studies 76, Princeton University Press, Princeton, 1974.

J. W. Vick, *Homology Theory: An Introduction to Algebraic Topology*, Graduate Texts in Mathematics 145, Springer, New York, 1994 (1st ed. Academic Press, 1973).