Geometry and Topology of Manifolds

2012-2013

Objectives

- To define orientability of topological manifolds and of smooth manifolds.
- To determine the multiplicative structure of singular cohomology.
- To calculate with differential forms on manifolds.
- To prove Stokes' theorem on manifolds and related results.
- To prove that de Rham cohomology coincides with real singular cohomology.
- To learn Poincaré duality and some of its consequences.
- To discuss the existence of nonzero sections of vector bundles on manifolds.

Evaluation of learning outcomes

Continuous assessment:

- Assessment of work done during problem sessions: 15%.
- Problem solving or small assignments: 35%.
- Final written exam: 50%.

Examination-based assessment is optional and will consist of a single written exam. All students are admitted to re-evaluation, which will also consist of a written exam. The final grade will be the highest score between evaluation and re-evaluation.

Teaching blocks

1. Singular cohomology. Homotopy invariance. Mayer-Vietoris exact sequence. Cellular complexes. Universal coefficients. Products.
2. Manifolds. Manifolds with boundary and without boundary. Smooth structures. Tangent bundle and cotangent bundle.
3. Cohomology of differential forms. Differential forms on manifolds. De Rham cohomology. Integration of forms with compact support. Stokes' Theorem. De Rham's Theorem.
4. Poincaré duality. Orientation classes. Compact support cohomology. Duality. Consequences and applications.

Reading and study resources

R. Bott, L. W. Tu, *Differential Forms in Algebraic Topology*, Graduate Texts in Mathematics, vol. 82, Springer, New York, 1986 (1st ed. 1982).

G. E. Bredon, *Topology and Geometry*, Graduate Texts in Mathematics, vol. 139, Springer, New York, 1993.

M. P. do Carmo, *Differential Forms and Applications*, Universitext, Springer, Berlin, 1994.

M. J. Greenberg, J. R. Harper, *Algebraic Topology: A First Course,* Mathematics Lecture Note Series 58, Addison-Wesley, Reading, 1981.

A. Hatcher, *Algebraic Topology*, Cambridge University Press, Cambridge, 2002.

K. Jänich, *Vector Analysis*, Undergraduate Texts in Mathematics, Springer, New York, 2001.

S. Morita, *Geometry of Differential Forms*, Translations of Mathematical Monographs, vol. 201, American Mathematical Society, Providence, 2001.

J. W. Vick, *Homology Theory: An Introduction to Algebraic Topology*, Graduate Texts in Mathematics, vol. 145, Springer, New York, 1994 (1st ed. Academic Press, 1973).