Geometry and Topology of Manifolds

2018-2019

Objectives

The course offers a combination of tools from differential geometry and algebraic topology aiming to solve problems involving manifolds. Concepts such as orientability and Poincaré duality are treated from both points of view. A proof of de Rham's theorem will be given, showing that cohomology of differential forms on smooth manifolds coincides with singular cohomology with real coefficients. Differential operators on smooth manifolds will be discussed in connection with the de Rham cochain complex.

Teaching Blocks

1. <u>Manifolds:</u> Topological manifolds. Examples. Smooth structures. Smooth functions. Partitions of unity. Tangent and cotangent spaces. Smooth maps between manifolds. Vector bundles. Smooth vector fields and smooth 1-forms.

2. <u>De Rham cohomology:</u> Multilinear algebra. Differential forms on smooth manifolds. Exterior derivative. The de Rham complex. De Rham cohomology. Poincaré lemma. Integration of differential forms on smooth manifolds. Integration along chains. Orientability. Stokes' theorem. Differential operators on smooth manifolds.

3. <u>Singular cohomology:</u> Homology and cohomology. Homotopy invariance. Kronecker duality. Mayer-Vietoris long exact sequence. Calculations with cell structures. Cup product. Local homology. Fundamental classes of compact orientable manifolds.

4. <u>Main results:</u> Proof of the de Rham theorem. Poincaré duality.

Reading and Study Resources

R. Bott, L. W. Tu, *Differential Forms in Algebraic Topology,* Graduate Texts in Math. vol. 82, Springer, New York, 1986 (1st ed. 1982)

G. E. Bredon, *Topology and Geometry,* Graduate Texts in Math. vol. 139, Springer, New York, 1993

M. P. do Carmo, *Differential Forms and Applications,* Universitext, Springer, Berlin, 1994

A. Hatcher, *Algebraic Topology,* Cambridge University Press, Cambridge, 2002

J. W. Vick, *Homology Theory: An Introduction to Algebraic Topology,* Graduate Texts in Math. vol. 145, Springer, New York, 1994 (1st ed. Academic Press, 1973)